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PROBLEMS OF PROCESSING COMPLEX SIGNALS IN CORRELATED SYSTEMS

V. I. Vinokurov, et al

Foreign Technology Division
Wright-Patterson Air Force Base, Ohio

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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

* ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

FOLLOWING ARE THE CORRESPONDING RUSSIAN AND ENGLISH

DESIGNATIONS OF THE TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}
rot	curl
lg	log

INTRODUCTION

Correlation methods are widely used in different branches of science and engineering, particularly in radio electronics, automatic systems, and control and measuring technology.

In measuring problems, which are based on the use of the correlation method, the correlator is of primary interest. In this case we study primarily the systematic error in correlators of different types operating without interference and under conditions where the secondary units of the system (amplifiers, filters, transducers, etc.) do not introduce additional error.

In examining systems for detecting and measuring the parameters of weak signals attention is focused on analyzing the potentials of the system for separating the useful signal in the presence of noise. Deviations from optimal design in the system which result from production technology are generally ignored.

In the monograph an attempt is made to study the effect of certain factors resulting from deviation from the ideal of the technical parameters of the system, which leads to a breakdown in correlation processing, on the parameters and noise characteristics of correlation systems. The first three chapters are dedicated to individual problems of the technical application of correlation methods. The characteristics of periodic noise signals are

examined, along with pseudonoise signals on a base of maximal (long) binary shift registration sequences. The properties of binary and ternary sequences are discussed. The results of analyzing the effect of nonlinear transformations of random Gaussian processes on the shape of the envelope for the correlation function of these processes are summarized. A brief survey is given on correlator construction methods. Discussed are the construction principles of correlators which instead of producing studied random processes form another function of these processes (functional correlators). A correlator with an ideal multiplier for a case where the studied random processes are narrow-band and have spectral densities concentrated in different frequency ranges (different frequency correlator) is analyzed. Simplified diagrams showing the construction of electronic correlators are presented. The action principle of the optical correlator is examined. Examined briefly in the third chapter are examples of correlation systems for different purposes: radar stations operating on noise and pseudonoise signals, passive systems which obtain information about the environment through receipt and correlation analysis of signals emitted by secondary sources, anti-fading communications systems based on the use of wide-band signals. Generalized block diagrams are given for the main types of correlation systems. These enable us to analyze the work of the system regardless of its purpose.

The fourth and fifth chapters analyze the work of functional correlators. It is assumed that additive noise enters the correlator together with the signal through one of the channels. The signal/noise ratio at the outlet of functional correlators with difference frequency, which forms the degree or exponential function of the inlet processes, are studied. The noise characteristics of functional correlators are compared with the characteristics of a correlator containing an ideal multiplier. Cases are studied in which the noise characteristics of the functional correlators are only slightly impaired as well as

functional dependences which greatly impair the noise characteristics. Discussed are the reasons for additional losses and ways in which they can be decreased.

Chapters 6 and 8 are dedicated to analyzing the effect of linear distortions in the spectral density of the studied processes on their cross-correlated and autocorrelated characteristics. Linear distortions appear in the absence of agreement between the frequency characteristic of the filter and the spectral density of the signal, when there is a scale shift in frequencies relative to the spectral density of the studied processes, etc. Linear distortions may be different in each of the investigated processes, which leads to a change in the cross-correlation function of these processes. Changes in the main lobe of auto- and cross-correlation functions and its shift along the delay axis, the increase in the side lobes, the deterioration in the signal/noise ratio, and other characteristics are examined. Noise and phase manipulated signals are analyzed. The distortions examined in the monograph by no means encompass all questions concerned with the development and construction of correlation systems. A number of questions not treated in the monograph are cited in the literature.

Numerous graphic representations of the investigated dependences are presented in this study. This, in the opinion of the authors, should facilitate understanding and use of the material in this book and, furthermore, make it possible to abbreviate some of the awkward mathematical transformations.

Questions of probability theory are not discussed in the monograph. A short list is given of the basic ideas on which the material is based. The material is discussed in relation to the problems of radioelectronics, although general methods and some obtained results may be extended to other branches of technology related to correlation analysis.

The results cited in the text frequently refer not to the work in which they were originally obtained, but in later works. Problems of priority require special analysis, and no analysis is made here. We realize that the literature presented in the monograph is incomplete.

The proposed book was intended for a wide circle of specialists involved in processing complex signals in the presence of noise and correlation systems and devices, as well as graduate students and students of senior courses specializing in the field of radioelectronics, control, and measurements.

The authors are extremely grateful to Professor S. I. Bychkov for his constant assistance in this work, his advice and criticism. We were also very grateful for the advice and comments by Professor Yu. Ya. Yurov, which he gave us in discussing individual phases of a work. The book was greatly improved by the valuable advice and comments given during editing by Professor I. N. Amiantov and Professor P. V. Olyanyuk, advice which the authors were very happy to receive.

The authors are grateful to the colleagues of the department of radio ship equipment of the LETI (ЛЭТИ) [Leningrad Electrotechnical Institute] for their creative discussion and help in formulating the manuscript.

The Authors

FIRST CHAPTER

CORRELATION PROPERTIES OF SIGNALS

1.1. SIGNIFICANCE OF CORRELATION METHODS IN ANALYZING RANDOM AND REGULAR PROCESSES

In science and technology a very important role is played by processes which are determined by a number of factors and interrelationships, detailed study of which requires the use of statistical analysis methods. The statistic approach refrains from predicting an exact result for each individual experiment and is based on the study of a group of experiments. With this approach one can discover regularities and quantitative relationships which can be described by means of the probability theory and mathematical statistics.

Random processes are often processes which occur in time. The mathematical apparatus of the theory of random functions is used for the characteristics of such processes. Function x(t) of argument t is called random if its value is a random quantity at any possible value of t [24, 37, 41]. Sometimes it is convenient to give a random function as an analytical formula in which certain parameters are random quantities (for example, in the form of trigonometric theories with random amplitudes and phases). It is standard practice to call x(t) a random (stochastic) process if argument t changes constantly. If t takes an even set of values,

then x(t) is called a random sequence. In observing a random process experimentally we are dealing with an individual realization or with a sample distribution function. A set of these realizations forms a random function. A random function is given if for any number of arbitrarily selected values $t_1, \ldots, t_n$ the n-dimensional density of the probability of this function $p_n(x_1, t_1, \ldots, x_n, t_n)$ is known. The greater the value of n, the more detailed will be the assignment of the random function.

In place of probability density the characteristic function, which is determined as the Fourier transform from probability density, may be assigned, i.e.,

$$\theta_n(u_1, t_1, \ldots, u_n, t_n) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p_n(x_1, t_1, \ldots, x_n, t_n) \times$$
$$\times e^{i(u_1 x_1 + \ldots + u_n x_n)} dx_1 \ldots dx_n. \tag{1.1}$$

Widely used in the theory of random functions are the characteristics of probability distribution, which are called the distribution moments. A particularly important role is played by the first and second moments, which determine the average value and correlation function, respectively, i.e.,

$$\overline{x(t)} = \int_{-\infty}^{\infty} x(t) p(x, t) dx,$$
$$K(t, \tau) = \overline{[x(t) - \overline{x(t)}][x(t-\tau) - \overline{x(t-\tau)}]} =$$
$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x_t - \overline{x}_t)(x_{t-\tau} - \overline{x}_{t-\tau}) p(x_t, x_{t-\tau}) dx_t dx_{t-\tau}, \tag{1.2}$$

where $p(x, t)$; $p(x_t, x_{t-\tau})$ are the one-dimensional and two-dimensional probability densities. The line above indicates averaging over a set of occurrences.

The correlation function between the values of a single random process which are separated from each other by time τ is called

the autocorrelation function. The correlation function, which can be determined for the values of two random processes x(t) and y(t) is called the cross-correlation function and is determined by the relationship

$$K_{xy}(t_1, t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x_1 - \bar{x}_1)(y_2 - \bar{y}_2)\times$$
$$\times p(x_1, t_1; y_2, t_2)\,dx_1\,dy_2, \qquad (1.3)$$
$$K_{yx}(t_1, t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(y_1 - \bar{y}_1)(x_2 - \bar{x}_2)\times$$
$$\times p(y_1, t_1; x_2, t_2)\,dy_1\,dx_2.$$

The correlation bond between the values of the two random processes at two different moments in time is described in the general case by the correlation matrix

$$K = \begin{Vmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{Vmatrix}.$$

Let us assume that the integrals in relationships (1.1)-(1.3) exist.

The correlation coefficient (standardized correlation function) also serves as the characteristic of random processes. It can be determined by the relationship

$$K_{н}(t_1, t_2) = \frac{K(t_1, t_2)}{\sqrt{K(t_1, t_1)\,K(t_2, t_2)}}. \qquad (1.4)$$

The correlation coefficient does not depend on the intensity of the studied random processes and can acquire values which lie in a range of from +1 to -1. The equality of the correlation coefficient to one indicates the presence of a functional dependence between the studied processes. The equality of the correlatic coefficient to zero indicates that the correlation relationship is absent. For random processes with a Gaussian distribution this denotes statistical independence. In the general case, in

non-gaussian distributions, the equality of the correlation coefficient to zero does not indicate the absence of a statistical connection between random processes.

Stationary processes represent an important class of random processes. A random process is stationary if its probability density $p_n(x_1, t_1, \ldots, x_n, t_n)$ of the arbitrary order n does not depend on the selection of a reference point for moments in time, i.e.,

$$p_n(x_1, t_1, \ldots, x_n, t_n) = p_n(x_1, t_1 + \tau, \ldots, x_n, t_n + \tau). \qquad (1.5)$$

The moment of a stationary random process depends only on differences $\tau_m = t_m - t_n$, while the average value is a constant quantity.

In solving many technical problems the mere knowledge of the average value and the correlation function is sufficient. The theory which considers the properties of the random processes which are determined by the moments of the first and second orders, i.e., by two-dimensional distribution, is called the correlation theory. Within the limits of the correlation theory random processes are considered stationary (in the broad sense) if their average value is constant, while the correlation function depends only on the difference in moments of time τ and acquires a final value when $\tau \to 0$.

Random processes which are strictly stationary [relationship (1.5)] will be stationary in the broad sense. Reverse confirmation is not valid.

Expressions for autocorrelation and cross-correlation functions of stationary random processes with a zero average value have the form

$$K(\tau)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xx_{\tau}p(x, x_{\tau})\,dx\,dx_{\tau},$$

$$K_{xy}(\tau)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy_{\tau}p(x, y_{\tau})\,dx\,dy_{\tau}. \qquad (1.6)$$

The autocorrelation function of a stationary random process is an even function, i.e., $K(\tau) = K(-\tau)$, acquires a maximal value when $\tau = 0$, and usually reverts to zero when $\tau \to \infty$. A decrease in the correlation function as τ increases does not exclude the possibility of peaks - side lobes. One characteristic of the correlation function (and random process) is the correlation time

$$\tau_k = \frac{1}{2K(0)}\int |K(\tau)|\,d\tau, \qquad (1.7)$$

which determines average value of the time interval in which the correlation dependence between the random process values occur.

Two random processes have a stationary relationship if their multidimensional combined probability density does not depend on the selection of the reference point for the moments in time. Within the limits of the correlation theory (stationarity in the broad sense) random processes have a stationary relationship if their cross-correlation function does not depend on the selection of the time reference point.

Stationary random processes which are encountered in practice almost always have an ergodic property. For ergodic random processes the stationary probability of state (i.e., probability with respect to a set of occurrences) is equal at the limit to the relative time of remaining in a given state.

The probability characteristics for ergodic random processes obtained by averaging a set of occurrences are equal with a probability at any distance from one to corresponding probability characteristic obtained from one occurrence of the random process

by averaging for a sufficiently large time interval. Thus, time averaged random process values found from individual occurrences of this process,

$$\langle x_i(t)\rangle = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\,dt$$

represent random quantities, but when $T \to \infty$ all within the limits tend toward the same quantity - the statistical average (i.e., to the average for the set of occurrences). This means that in determining the average values it is not necessary to consider the many occurrences of the random process; it is sufficient to study one occurrence existing for a prolonged time. The conditions under which the indicated property will occur are called the conditions of ergodicity. For ergodic random processes the autocorrelation function can be found on the basis of one event in the process

$$R(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x(t)\,x(t-\tau)\,dt, \qquad (1.8)$$

where $R(\tau) = K(\tau)$ with a probability of one. Similarly the cross-correlation function can be determined on the basis of a single event of the stationary random processes

$$R_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x(t)\,y(t-\tau) \qquad (1.9)$$

If, however, these processes together form an ergodic process, then, with a probability equal to one,

$$R_{xy}(\tau) = K_{xy}(\tau).$$

The ergodic property is extremely important in practice. Experimental determination of the characteristics of random

processes is generally done on the basis of a single occurrence of the process. The obtained characteristics are regarded as statistical characteristics. In mathematical analysis of random processes we use statistical averaging (for a set of occurrences), although the phenomena occur in time and represent separate occurrences of the studied process. In all such cases the ergodic property is presumed to be valid.

Correlation methods are also significant in that the correlation function is related by the integral Fourier transform to the spectral characteristics of the random process [24, 37, 41]. For a stationary random process this relationship is determined by the Wiener-Khinchin theorem:

$$R(\tau)=\int_{-\infty}^{\infty} S(f)\,e^{i2\pi f\tau}\,df; \quad S(f)=\int_{-\infty}^{\infty} R(\tau)\,e^{-i2\pi f\tau}\,d\tau, \tag{1.10}$$

where $S(f)$ is the spectral density of the random process.

For nonstationary processes which have finite power, i.e.,

$$\lim_{T\to\infty}\frac{1}{T}\int_{0}^{T} x^2(t)\,dt<\infty$$

(the energy of the signal may be infinitely great), relationships (1.10) are valid for the time averaged value of the correlation function determined by the formula [26]

$$R(\tau)=\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} \dot{K}(t,\tau)\,dt.$$

F[illegible]n processes having finite energy,

$$\int_{-\infty}^{\infty} x^2(t)\,dt<\infty,$$

the relationships of (1.10) are valid for the function $R(\tau)$, which is determined from formula

$$R(\tau)=\int_{-\infty}^{\infty} K(t, \tau)\,dt.$$

In this case $S(f)$ represents the spectral density of signal energy.

The relationships of (1.10) are also valid in studying the cross-correlation function. However, in this case $R_{xy}(\tau) \neq R_{xy}(-\tau)$, while the energy spectrum $S_{xy}(f)$ becomes a complex function.

Correlation dependences play an important role in analyzing the properties and characteristics of regular processes as well as random.

In problems in which the parameters of motion are measured the signal which is reflected from the target is distinguished from the emitted signal not only by its time shift but by its frequency shift, which is acquired by the Doppler effect. The properties of the signal in this case are described by the generalized correlation function (the function of uncertainty), determined by the relationship [[illegible], 8, 39]:

$$R(\tau, f)=\int_{-\infty}^{\infty} x(s)\,x^{*}(s-\tau)\,e^{i2\pi fs}\,ds, \qquad (1.11)$$

where $x(t)$ is the complex modulating function which considers the amplitude and frequency modulations of the probing signal; $x^{*}(t)$ - the complex-conjugate function; τ - the difference between the actual and expected signal delay; f - the difference between the actual and expected Doppler frequency shift. The generalized correlation function can be standardized, i.e., $\rho(\tau, f) = R(\tau, f)/R(0, 0)$. In detection problems the modulus of functions $|R(\tau, f)|$ or $|\rho(\tau, f)$ is examined. In the rectangular coordinate system of ρ, τ, f the uncertainty function is expressed

in the form of the surface. A body bounded by this surface and the coordinate plane $\rho = 0$ is called a body of uncertainty. The shape of the body of uncertainty is solely dependent on the shape of the signal. The best signal for measuring the parameters of motion of the target would be a signal whose body of uncertainty had one narrow peak at the origin of the coordinate and the least side lobe level at other values of τ and f. Systems which transmit information under multi-beam conditions have similar signal requirements. Analyzing the uncertainty (or its square) makes it possible to determine the range and rate revolutions of a given signal, estimate measurement accuracy, etc.

The change in the emitted signal may be caused not only the Doppler effect but also by a change (frequently static) in the properties of the medium in which the electromagnetic waves are propagated. Such changes occur, for example, in a case where the antenna is surrounded by a plasma whose temperature and turbulence are different over the aperture. Attempts have been made [76] to use correlation relationships for analyzing the operational peculiarities of radio units taking into account the properties of the space in which the electromagnetic waves are propagated. For this reason the "transcorrelation function" was introduced, which is the time and space averaged correlation function of the electromagnetic fields. The space in which the radiowaves are propagated is treated as a system which has a transmission function and which connects an undistorted wave front at the inlet (transmitter) to a distorted wave front at the outlet (receiver). The correlation properties of the electromagnetic fields are related to the possibility of transmitting information by means of these fields. Thus, the additions of the maximum transcorrelation functions describe a system with a configuration in which information transmission conditions will be optimal (for example, optimal distribution of the field in the aperture of an antenna working in an ionized medium).

Correlation analysis is used to study the properties of a medium in which mechanical vibrations are propagated [38]. For example, the studied space of an ocean may be regarded as a certain system in which an acoustical signal is transmitted to the "input" and the response is measured at the "output" (i.e., at another point in the ocean). The correlation connection between the "output" and "input" effects can be found. The positions of the "input" and "output" in the medium are diverse and the measurement results for different cases can be different. Analysis of correlation dependences for input and output signals given a finite number of different spatial measurements can be the material which is used to study and simulate the propagation conditions of sound vibrations in a medium and the properties of the medium itself.

Correlation relationships are very important if we are studying systems in which information is transmitted [4, 18]. The signals which carry different information can be regarded as vectors in Hilbert space. Vector characteristics include the norm (length of vector) and distance between vectors. For a signal which exists in a time interval of $0 \leq t \leq T$ the norm is equal to the square root of energy. The square of the distance between the vectors when signal energies are the same is expressed as their cross-correlation coefficient

$$d_{ij}^2 = 2E(1 - \rho_{ij}). \qquad (1.1[illegible])$$

where E is signal energy; ρ_{ij} is the cross-correlation coefficient of the signals.

Analysis shows [4, 18, 81] that the greater the signal energy and distance between the vectors, the smaller the probability of error in receiving the communication. In the case of limited signal energy, the greater the factor $1 - \rho_{ij}$, the greater will be the distance between vectors. For two signals the best will be

those in which $\rho_{ij} = -1$. In the presence of a great number of signals the best value of ρ_{ij} is close to zero.

Thus, by means of analyzing the cross-correlation coefficient we can determine how close the studied signals or correlation codes are to the optimal.

The examples given above are far from comprehensive in regard to the application of the correlation function in radio-electronics. Correlation analysis plays a very significant role in systems with variable parameters, in the theory of object discrimination, in solving many automatic regulation and control problems [23, 34, 37], etc.

1.2. PERIODIC AND REPEATED PROCESSES

A function is called periodic if we can find for it a certain quantity T_0 for which the condition $f(t) = f(t + T_0)$ is fulfilled for any value of argument t. The least value of T_0 is called a period.

Processes which are dealt with in radioelectronics exist in a finite time segment. In a number of cases during the time that the vibration exists the number of repetitions can be great and the correlation properties of the studied process can be close to the properties of the corresponding periodic process. In cases where the number of repetitions is small, the correlation properties of the process change substantially.

The correlation function of a periodic process is determined by formula (1.8), where x(t) is the periodic time function. For a harmonic vibration with a constant amplitude A, frequency ω, and arbitrary initial phase

$$R(\tau) = \frac{A^2}{2}\cos\omega\tau. \qquad (1.13)$$

A complex periodic vibration can be represented in the form of the Fourier theory. In this case the expression for the correlation function will have the form of

$$R(\tau) = \frac{a_0^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty} a_n^2 \cos n\omega\tau, \tag{1.14}$$

where a_0 and a_n are the amplitudes of the harmonic component.

From the obtained relationships it follows that the correlation function of the periodic process is periodic with the same period as in the time function, and does not contain information on the phase of the studied process.

The cross-correlation function of two periodic processes is found by formula (1.7) and contains only components of those frequencies which are present in both processes. The cross-correlation function contains information on the phase of the components of the same frequency in the studied processes.

The periodicity feature of the correlation function can be used to detect a weak periodic signal against a background of noise [61].

Let us examine a signal which repeats within a limited time interval a voltage with a complex shape, for example, a pulse sequence. Let us assume that the pulses are separated by equal time intervals, forming an even sequence. The length of the pulses is less than half of the interval between them.

Figure 1.1 shows the correlation function of an even pulse sequence. The greatest peak corresponds to a zero delay time. When the delay changes, the peak amplitude decreases according to the law $1 - |m|/N$, where m is the integer of pulse repetition intervals contained in delay τ; N is the number of pulses in the sequence; $m \leq N$.

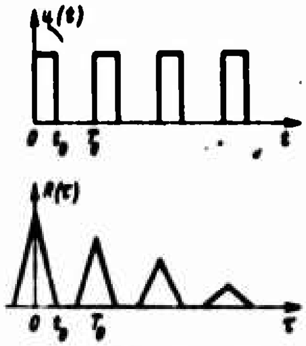

Fig. 1.1. Correlation function of uniform pulse sequence.

The decreased amplitude of the spikes is explained by the fact that when the delay is changed for the repetition interval the number of pulses participating in the formation of the correlation integral in each sequence is decreased.

The correlation function of a signal consisting of one pulse has only one maximum at $\tau = 0$, and reverts to zero when $|\tau| > t_0$ (where t_0 is the length of the pulse).

The repetition of pulses in time is the reason for the appearance of repetition with respect to τ in the correlation function.

The uncertainty function of a uniform sequence of pulses when $f = 0$ coincides with the correlation function (Fig. 1.1). In section $\tau = 0$ the uncertainty function has the greatest side spikes at points $f = q/T_0$ (Fig. 1.2), where $q = 0, 1, 2, \ldots$; T_0 is the interval between pulses in a uniform sequence.

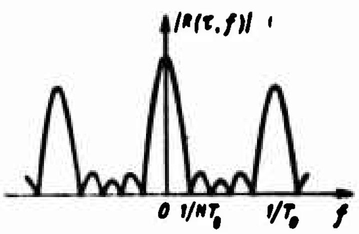

Fig. 1.2. Section of uncertainty function in uniform pulse sequence ($\tau = 0$).

The appearance of several spikes for the uncertainty function in a uniform pulse sequence indicates uncertainty in determining distance to the target and speed (if additional limitations are not used, for example, $\tau_{max} < T_0$; $f_{max} < 1/T_0$).

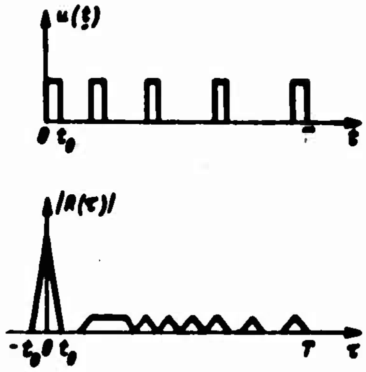

Fig. 1.3. Correlation function of pulse sequence with linearly ascending interval.

The side lobes of the uncertainty function can be greatly reduced if the repetition of pulses in a sequence at equal time intervals is eliminated. If the interval between pulses in the sequence is increased linearly, then conditions can be created under which all impulses of main and delay sequences [formula (1.8)] will coincide only in a zero time shift. When $\tau > t_0$ they will coincide only in one pulse of the main and delay sequences. Figure 1.3 shows the typical appearance of the correlation function of pulse sequence with a linearly ascending repetition interval. In this case the level of side lobes has been decreased N times compared to the main maximum.

There are other methods of decreasing the level of side lobes in the uncertainty function. These are based on removing the periodicity of the pulses in the sequence, and can be achieved, for example, in the Sherman sequence [37].

1.3. RANDOM AND PSEUDORANDOM PROCESSES

1. Random Processes

In many processes which are dealt with in radioelectronics the probability density has a gaussian distribution law. Such processes include, for example, thermal noise, fluctuation, and useful signals in a number of cases [77]. The prevalence of gaussian random processes can be explained by the central limiting theorem of the probability theory, according to which the sum of a great number of random processes having arbitrary distributions has a normal probability density distribution at the limit

(additional conditions requiring that each of the terms making up the sum have a small, uniform effect are not rigid).

A multidimensional normal distribution function depends only on the average value and the values of the correlation coefficients. This means that the correlation function and the average density value determine the normal random process. For a nonstationary random process the average value and the correlation function depend on time.

In order for a stationary gaussian process to have the ergodic property it is sufficient that its energy spectrum be continuous, i.e., that the following integrals converge [24]:

$$\int_{-\infty}^{\infty} |K(\tau)|\, d\tau < M; \quad \int_{-\infty}^{\infty} |S(f)|\, df < P, \tag{1.15}$$

where M and P are finite constant values.

From the spectral density shape wide-band and narrow-band random processes can be distinguished. For the studied narrow-band random processes the frequency band which is occupied by spectral density is much smaller than the average frequency. A narrow-band process can be conveniently described by a random function in the form of

$$x(t) = E(t)\cos[\omega_0 t - \varphi(t)], \tag{1.16}$$

where E(t), ϕ(t) represent the envelope and phase of the random process.

This representation is based on the possibility of determining random process x(t) by means of the conjugate η(t) process, which is related to the original Hilbert transform:

$$x(t) = \frac{-1}{\pi} \lim_{T \to \infty} \int_{-T}^{T} \frac{\eta(\tau)}{\tau - t} d\tau,$$

$$\eta(t) = \frac{1}{\pi} \lim_{T \to \infty} \int_{-T}^{T} \frac{x(\tau)}{\tau - t} d\tau.$$

In the case of $E(t) = \sqrt{x^2(t) + \eta^2(t)},$

$$\Phi(t) - \omega_0 t - \varphi(t) = \operatorname{arctg} \frac{\eta(t)}{x(t)}.$$

Two-dimensional probability density of the envelope and random phase of the gaussian process is determined by the formula

$$p(E, E_\tau, \varphi, \varphi_\tau) = \frac{EE_\tau}{4\pi\sigma^4(1-\rho^2)} \times \\ \times \exp\left\{ \frac{-1}{2\sigma^2(1-\rho^2)} [E^2 + E_\tau^2 - 2\rho EE_\tau \cos(\varphi_\tau - \varphi)] \right\}, \quad (1.17)$$

where σ is the mean square value of the random process.

The autocorrelation function of the narrow-band random process is determined by the relationship

$$R(\tau) = \sigma^2 [r_c(\tau) \cos \omega_0 \tau + r_s(\tau) \sin \omega_0 \tau] = \\ = r(\tau) \cos[\omega_0 \tau + \zeta(\tau)], \quad (1.18)$$

the factors $r_c(\tau)$ and $r_s(\tau)$ have the form of

$$\left. \begin{matrix} r_c(\tau) \\ r_s(\tau) \end{matrix} \right\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega_0 - \omega) \begin{matrix} \cos \omega t \\ \sin \omega t \end{matrix} \, d\omega,$$

$$r^2(\tau) = r_c^2(\tau) + r_s^2(\tau), \quad \zeta(\tau) = \operatorname{arctg} \frac{r_s(\tau)}{r_c(\tau)}.$$

Since the spectral density of the random process is concentrated in a narrow frequency band near ω_0, function $S(\omega_0 - \omega)$ is maximal in the low frequency range. If the spectral density can be

considered symmetrical in relation to frequency ω_0, then $r_s(\tau) = 0$ (since we have the product of the even times the uneven function under the integral sign) and, consequently, $r(\tau) - r_s(\tau)$; $\zeta(\tau) = 0$. The expression for the correlation function in this case will be

$$R(\tau) = \sigma^2 r(\tau) \cos \omega_0 \tau. \qquad (1.19)$$

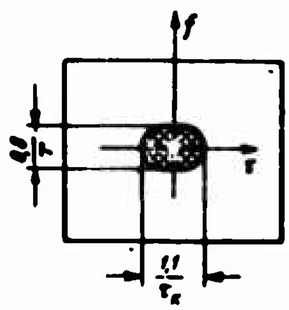

Fig. 1.4. Section of uncertainty function in noise signals.

The correlation function of a random process has only one maximum when $\tau = 0$ and is close to zero when $|\tau| > \tau_k$. The body of uncertainty of a signal has one high correlation range when $\tau = f = 0$ against a background of a field of low intensity spikes (Fig. 1.4).

3. Pseudorandom Processes

The sharp maximum and low level of side lobes in the body of uncertainty are possessed not only by sound signals, but also by signals formed by amplitude manipulation and by frequency or phase modulation of a carrier within a pulse according to a certain law. In systems with frequency modulation the linear law of frequency change is widely used. The section of the body of uncertainty of a signal with linear frequency modulation formed by a horizontal plane has an elliptical shape, and its axes are turned at a certain angle relative to the Y-axes. Signals with linear frequency modulation do not have properties which are similar to those of random processes.

To achieve amplitude- or phase-manipulated signals the discrete law of change in the appropriate parameter is widely used. In this case a signal of length T would consist of N identical

positions - symbols, on each of which a harmonic oscillation with an amplitude of A_i and phase ϕ_i is formed. Sets of values $A_1, \ldots, A_n, \phi_1, \ldots, \phi_n$ form codes, which are selected such that they assure the necessary uncertainty function (and, above all, a low side lobe level). The difficulties of achieving signals with a great diversity of A_i and ϕ_i have led to the widespread use of signals with stepped phase changes for identical amplitudes or stepped amplitude changes when there is no change in phase. In most cases the phase change is by quantity π.

Phase- or amplitude-manipulated signals can conveniently be written as a sequence of symbols by making +1 correspond to an oscillation with an amplitude of one and a zero phase and making -1 (or zero) correspond to an oscillation with an amplitude of one and phase π. The correlation properties of the signal are determined by the type of binary sequence controlling the phase (or amplitude) change. In the following discussion attention will be focused on phase-manipulated signals.

A low level of side lobes in the uncertainty function can be provided by the Barker codes (not more than $1/N$, where N is the number of code symbols). Barker sequences are known for $N \leq 13$, which limits their use. Currently there are codes of greater duration which have a side lobe level and the uncertainty function of no more than $1/\sqrt{N}$. For example, binary m-sequences, technically rather simple to achieve through shift registers, are widely used.

The shift register is a device consisting of cascade-connected binary storage elements. The number of storage elements determines the configuration of the register. Under the influence of the clock pulses, which are supplied to the register from a separate oscillator, the electrical state of the binary cells shifts (moves) across the register to (successive) elements. The sequence of voltage values, which is set at the register output, depends on the initial state of the cells. The length of one symbol of the code being formed is determined by the repetition

period of the clock pulses. The length of the entire sequence, which can be described by the number of symbols, is equal to the number of cells in the register, i.e., N = k.

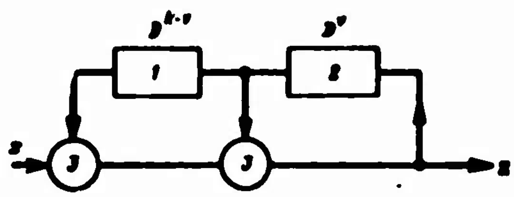

Fig. 1.5. The simplest block diagram of a binary sequence generator: 1, 2 - shift register cells, which provide delay for k - ν and ν intervals; 3 - logic feedback element which adds modulo 2.

For a given number of cells considerably longer sequences can be obtained if logic feedback is introduced. In the feedback element the logic function of the state of several cells is calculated, and the result is fed to register input. Units with feedback can be characterized by an operator with delay D corresponding to a delay of one [cycle] unit. A delay of ν units corresponds to operator D^{ν}. The work of such a system, shown in Fig. 1.5, can be characterized by an operator such as

$$z = x \oplus D^{k} z \oplus D^{\nu} z, \tag{1.20}$$

where ⊕ denotes adding modulo two (Table 1.1); x and z are the input and output sequences.

Table 1.1. Adding modulo two.

Terms		Results
0	1	1
1	0	1
0	0	0
1	1	0

In an autonomous regime, where from a certain time the signal is not obtained (x = 0), the operator has the form of

$$D^{k} z \oplus D^{\nu} z \oplus z = 0.$$

The delay operators can be used to represent the work of systems containing several logic feedbacks.

For the selected k-bit of the register feedbacks can be introduced in different ways. Yet not all feedbacks will provide the longest sequence.

The greatest sequence length which can be obtained is equal to $N = 2^k - 1$. As the clock pulses continue the sequence will be repeated. Sequences lasting $2^k - 1$ are called maximal length sequences or m-sequences, and they have properties similar to those of random binary processes. The polynomials which determine the formulation of binary m-sequences have been found for a number of cases [32], and some of them are given in Table 1.2.

Table 1.2. Delay polynomials.

k	Polynomial	Length
3	$D^3 \oplus D \oplus 1$	7
4	$D^4 \oplus D \oplus 1$	15
5	$D^5 \oplus D \oplus 1$	31
6	$D^6 \oplus D \oplus 1$	63
7	$D^7 \oplus D^3 \oplus 1$	127
9	$D^9 \oplus D^4 \oplus$	511
10	$D^{10} \oplus D^3 \oplus$	1023

The systems of the shaping unit correspond to the scheme shown in Fig. 1.5. The values of k and v should correspond to the type of polynomial selected (Table 1.2).

Binary m-sequences have interesting properties; the main ones are presented below.

1. In the binary m-sequence symbols +1 and 0 are often practically the same. For a sequence of any period the number of symbols of a single sign can differ from the number of symbols of the other sign by only one.

2. In a single realization of the sequence half of all series +1 and 0 have a length of one symbol, one quarter have a length of two symbols, one eighth - of three symbols, etc.

3. If the sequence is compared to its cyclical time shift for a certain number of symbols, then the number of corresponding symbols differs from the number of noncorresponding symbols by no more than one. In the case of a zero shift all symbols correspond.

4. The sum (modulu two) of two sequences which are time shifted by any number of [cycles] units (within the limits of the period) represent a sequence of the same polynomial but with a different time displacement. The given property is the basis of the sequence storage systems.

5. The normalized correlation function of a binary m-sequence is determined on the basis of formula (1.8) and has the form of a broken line.

Let us now dwell on the correlation properties of the sequences. Discrete time shifts can be easily studied. The expression for the correlation function in this case has the form of

$$\rho(j, t_0) = \frac{1}{N} \sum_{i=1}^{N-j} x(i)\, x(i+j). \tag{1.21}$$

As the shift increases in time the number of terms in the sum of (1.21) decreases.

The correlation function has its maximal value at zero time shift ($j = 0$). At shifts of $1 \leq j \leq N - 1$ there are side lobes, whose maximal level for great sequences ($N > 13$) does not exceed $1/\sqrt{N}$. For $N \leq 13$ there are m-sequences for which the side lobe level of the correlation function does not exceed $1/N$, as for the Barker code, i.e.,

$$\rho(j) = \begin{cases} 1/N & \text{при } N < 13, \\ 1/\sqrt{N} & \text{при } N > 13. \end{cases} \tag{1.22}$$

[при = when]

Figure 1.6 shows the form of the correlation function of a binary m-sequence for the particular case.

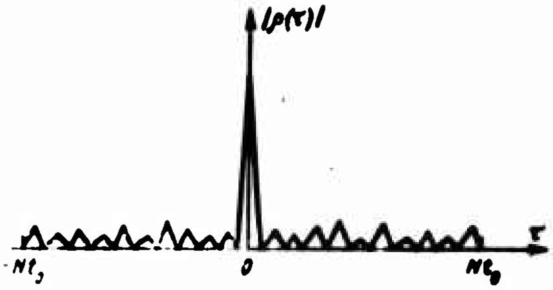

Fig. 1.6. Correlation function of m-sequence (+1, -1).

In a number of technical problems signals formed by periodically repeated m-sequences are used with an averaging time equal to the period. The correlation function of such a signal is determined for discrete delays by the relationship

$$\rho(j) = \frac{1}{N}\sum_{i=1}^{N} x(i)\,x(i+j). \qquad (1.23)$$

According to the properties of 3 and 4, as a result of the summations in formula (1.21) we get the m-sequence of the same polynomial, in which the number of symbols with different signs differ from one another by one. Thus

$$\rho(\tau) = \begin{cases} 1 - \dfrac{|\tau|(N+1)}{N t_0} & \text{при } |\tau| < t_0, \\ -\dfrac{1}{N} & \text{при } t_0 < |\tau| < t_0(N-1). \end{cases} \qquad (1.24)$$

[при = when]

The correlation function of the periodic m-sequence has repeated spikes (Fig. 1.7) and a side lobe level of 1/N.

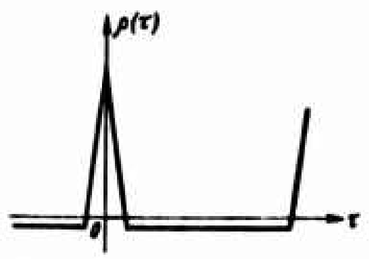

Fig. 1.7. Correlation function of periodic m-sequence.

The spectral density of the sequence is found as the Wiener-Khinchin transform from the autocorrelation function. The spectral density envelope and the position of its zero are determined by the shape and length of the symbol. The first zero of the envelope corresponds to a frequency of $1/t_0$. The spectral density of the periodic

sequence has a discrete nature. The frequency interval between neighboring spectral lines depends on the length of the sequence and is equal to $1/Nt_0$. In the case of a single sequence the spectrum becomes solid, and the shape of the spectral density envelope is preserved.

The uncertainty function of a phase-manipulated signal has one maximum at the origin of the coordinates, and in the section of plane **f** = 0 it repeats the correlation function. At different nonzero values of τ and f the uncertainty function has side lobes whose magnitude hardly exceeds $1/\sqrt{N}$. The form of the uncertainty [illegible] of a pseudorandom signal consisting of seven symbols is [illegible] Fig. 1.8.

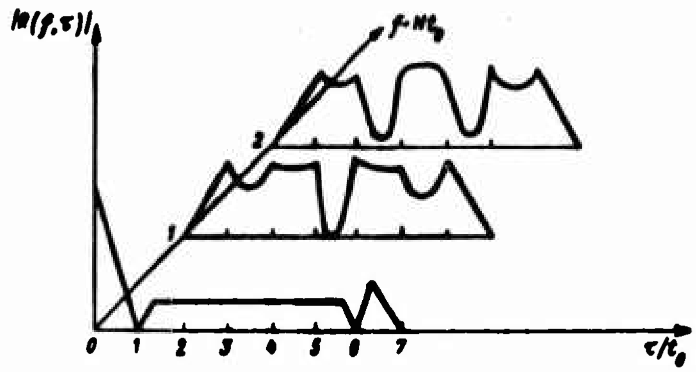

Fig. 1.8. Uncertainty function of phase-manipulated signal (N = 7).

M-sequences are called pseudorandom, which points to their similarity to random sequences. A random sequence is a sequence for which only the appearance of a certain symbol or group of symbols can be shown. The M-sequence belongs to regular signals, which are formed on the basis of coding rules. Its regularity lies in the fact that under known initial conditions its values can be calculated and predicted for a given moment in time. If, however, we compare binary m-sequences, for example, with a random sequence, then we notice a similarity between their structures. The similarity appears, for example, in the frequency of the different symbol combinations (properties 1 and 2) and in the form of the autocorrelation function.

For the purposes of autocorrelation processing the possibility of delaying pseudorandom signals is very important. In the case of an m-sequence delay can be achieved by some complication of the shaping systems. A delayed binary sequence may be obtained, for example, by taking voltage of various shift register cells. The number of delayed sequences in this case is determined by the configuration of the register, and does not include all possible signals in time. To obtain sequences corresponding to missing delays an additional device must be introduced which forms logic functional operations over sequences obtained at the various register cell outputs [70, 81]. The action of this device is based on property 4: the sum (modulo two) of two m-sequences of a given polynomial is also the m-sequence of the same polynomial [20, 46]. The logic system of obtaining delayed sequences is shown in Fig. 1.4. By combining the terms (i.e., voltages from the register cells) and adding modulo two, sequences can be obtained which are time shifted by different numbers of units (within the limits of the period) in relation to the output.

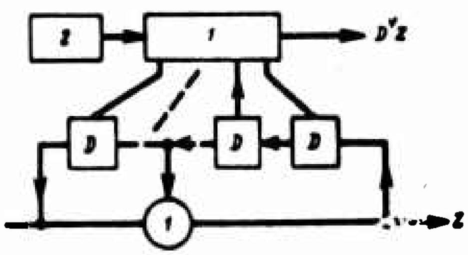

Fig. 1.4. Block diagram representing shaping of delayed sequence: D - register cells, 1 - modulo two adding system, 2 - delay control block.

The sequences discussed above are far from comprehensive with respect to the great diversity of binary sequences with side lobe levels of $1/\sqrt{N}$ [37].

Binary sequences with a large number of symbols can be obtained by combining short m-sequences [12]. The elements of the combined sequence are formed by logic operations performed on elements of the original sequences. An example might be a combined sequence formed according to the law $w = x \oplus yz$, where x, y, and z are binary sequences of shorter duration. If the periods making up the

sequences do not have a common denominator, then the period of the combined sequence is equal to the product of the periods of the components, i.e.,

$$N_{kr} = N_a N_b N_c.$$

The autocorrelation function of a combined sequence has spikes at all delay values which are divisible by the period of any of the component sequences. The greatest spike corresponds to zero delay; additional spikes have a smaller value. Combined sequences are used in systems for measuring the distance to an object, and make it possible to reduce the time of determining the phase of a reflected signal.

Ternary sequences with levels of +1, 0, -1 [80] have interesting correlation properties. The symbol 0 corresponds to the absence of oscillations (amplitude zero), +1 and -1 correspond to oscillations with an amplitude of one and an initial phase of zero and π, respectively. A ternary m-sequence can be described by delay operators adding modulo three. The number of symbols in a ternary sequence of maximal length is determined by the formula $N = 3^k - 1$. The autocorrelation function of a periodic ternary m-sequence has a positive maximum at a zero time shift and is negative for a shift of half a period (Fig. 1.10). In other delays of greater symbol duration the correlation function is equal to zero. The presence of a negative lobe in the correlation function is not desirable, particularly for signals which are used in detection systems.

From ternary m-sequences Chang found sequences of nonmaximal length [80] whose periodic correlation function had only one maximum when $\tau = 0$ and zero lobes for other delays. The length of the sequence is determined by the relationship

$$N_{\text{н}} = \frac{1}{2} N = \frac{1}{2}(3^k - 1).$$

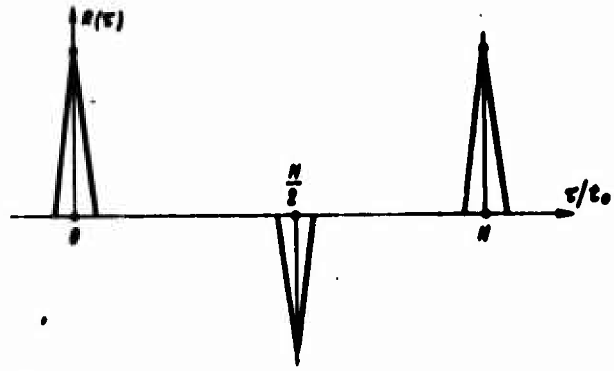

Fig. 1.10. Correlation function of ternary periodic m-sequence.

There are generators for ternary sequences of length N_4 whose correlation function has zero side lobes [80]. Table 1.3 shows some of the polynomials which determine the shaping systems of such sequences.

Table 1.3. Delay polynomial of ternary sequences.

k	Полином	Длительность
3	$D^3 \oplus 2D \oplus 2$	13
5	$D^5 \oplus 2D \oplus 2$	121
7	$D^7 \oplus D^3 \oplus 2$	1093

The properties of the ternary sequences of Chang are similar to those of the binary sequences studied above. For example, the number of +1 and -1 symbols in the sequence differ very little, particularly when the length of the sequence is increased (Table 1.4).

Table 1.4. Frequency of symbol occurrence.

k	N	m_{+1}	m_{-1}
3	13	6	3
5	121	45	36
7	1093	378	351

The symbols used in Table 1.4 are:

m_{+1} - the number of symbols in a sequence corresponding to +1;

m_{-1} - the number of symbols corresponding to -1.

A ternary sequence can be time delayed by adding modulo three sequences of a single polynomial which, however, are time-shifted. For the Chang sequence which consists of 13 symbols the methods of attaining time delay are determined by operators such as [80]

$$D^3x = Dx \oplus x; \quad D^4x = D^3x \oplus x; \ldots D^{11}x = D^3x \oplus 2x,$$

where $\oplus$ denotes addition modulo three.

The studied ternary sequences are interesting for a number of correlation electronics problems, since their correlation func is the closest to the correlation function of a single pulse a at the same time, they can be used to assure the given energy spectrum distribution on a large time interval, which is essential in detection and communications problems.

1.4. CORRELATION PROPERTIES OF GAUSSIAN RANDOM PROCESSES IN NONLINEAR TRANSFORMATIONS

Nonlinear transformations of random processes can be expect to lead to a change in the cross-correlation function. The algorithm for the work of the correlator according to formula (1.9) can be written as

$$R(\tau) = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} x(t_i)\, y(t_i + \tau).$$

Instrument readings at the system output (Fig. 1.11) depend on the correlation of input random processes. The greater the coincidence probability for the values of processes shifted relative to one another in time τ, the greater will be the readings. The sign of the terms of the sum is determined by the polarity of the co-factors. When $R(\tau) \to 0$ the polarity of the co-factors on the average will be identical for half of the terms and opposite for the remaining terms. The sums of positive and negative terms are equal on the average. In the case of nonzero $R(\tau)$ the weight of terms with the same or opposite co-factor polarity increases. Corresponding to the maximal value of $R(\tau)$ is the maximal quantity by which the sum of terms of one sign exceeds the sum of another sign. For the autocorrelation function

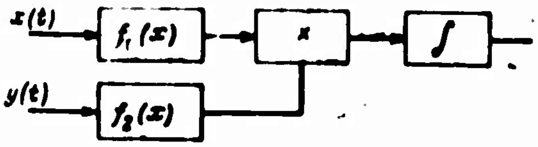

Fig. 1.11. System for analyzing nonlinear distortions.

when $\tau \to 0$ the polarity of co-factors corresponds (or is opposite), and the absolute value of the terms are maximal.

In the case of nonlinear transformations the terms represent the product of functions of random process values, i.e.,

$$f_1[x(t_1)]f_2[y(t_1+\tau)].$$

For most transformations polarity agreement among the co-factors will influence the magnitude of the sum just as if the transformation were absent, although the absolute values of the co-factors will change, thus leading to a change in the values of the correlation function.

Now let us examine the effect of nonlinear transformations on the correlation function of random processes with zero average values. A dependence must be established between correlation functions before and after the transformation [24, 45, 50].

Let us assume that extraneous noise and interference are absent; the ergodic property is valid.

We designate:

$x(t)$, $y(t)$ as random processes of the correlator input;

$K(\tau)$ - the correlation function between inlet processes (prior to nonlinear transformations);

$\psi(\tau)$ - the correlation function of random processes after nonlinear transformation.

Nonlinear transformations in the system (Fig. 1.11) are described by function $f_1(x)$, $f_2(y)$.

The main stage of the study is proving the validity of the relationship

$$\frac{\partial^k \psi(\tau)}{\partial K^k(\tau)} = \overline{f_1^{(k)}(x) f_2^{(k)}(y)}, \qquad (1.25)$$

where $f^{(k)}(x)$ is a k-th order derivative from function $f(x)$.

Proof consists essentially of the following. Voltages at the output of each nonlinear unit can be represented by means of the Laplace transform in the form of

$$f_i(x) = \frac{1}{2\pi i}\left\{\int_{C_{i+}} h_{i+}(u)\, e^{iux}\, du + \int_{C_{i-}} h_{i-}(u)\, e^{iux}\, du\right\}, \qquad (1.26)$$

where C_{i+}, C_{i-} are the corresponding integration circuits; $h_{i+}(u)$ is the Laplace transform from the system characteristics, which is equal to

$$h_{i+}(u) = \int_0^{\infty} f_i(x)\, e^{-iux} dx,$$

$$h_{i-}(u) = \int_{-\infty}^{0} f_i(x)\, e^{-iux} dx. \qquad (1.27)$$

If we substitute (1.26) in (1.6) and change the order of integration, then the expression for the correlation function can be represented in the form of the sum of integrals

$$\psi(\tau) = \left(\frac{1}{2\pi i}\right)^2 \times$$

$$\times \sum_{\pm} \int_{C_{1\pm}} du_1 \int_{C_{2\pm}} h_{1\pm}(u_1)\, h_{2\pm}(u_2)\, \varphi(u_1, u_2)\, du_2. \qquad (1.28)$$

where $\theta(u_1, u_2)$ is the characteristic function of the second order.

All possible combinations of $\pm$ signs are added. The left part of formula (1.25) when k = 1, according to (1.28) equals

$$\frac{\partial \psi(\tau)}{\partial K(\tau)} = \left(\frac{1}{2\pi i}\right)^2 \sum_{\pm} \int_{C_{1\pm}} du_1 \int_{C_{2\pm}} h_{1\pm}(u_1) h_{2\pm}(u_2) \frac{\partial \theta(u_1, u_2)}{\partial K} du_2. \qquad (1.29)$$

Now let us find the right part of (1.25) by using (1.26). We get:

$$f_1^{(1)} f_2^{(1)} = \left(\frac{1}{2\pi i}\right)^2 \sum_{\pm} \int_{C_{1\pm}} du_1 \int_{C_{2\pm}} h_{1\pm}(u_1) h_{2\pm}(u_2) \times \theta(u_1, u_2) u_1 u_2 du_2. \qquad (1.30)$$

Analysis shows that the equality of (1.29) and (1.30) takes place under arbitrary quantities h_1, h_2, i.e., under arbitrary functions of $f_i(x)$, and when the following condition is valid:

$$\frac{\partial \theta(u_1, u_2)}{\partial K(\tau)} + u_1 u_2 \theta(u_1, u_2) = 0. \qquad (1.31)$$

i.e., the equality is valid for random processes whose characteristic function satisfies equation (1.31).

Let us find the expression for the characteristic function. The solution to equation (1.31) has the form of

$$\ln \theta(u_1, u_2) = -u_1 u_2 K(\tau) + G, \qquad (1.32)$$

where G is the function subject to determination.

To determine function G we will use the initial conditions:

a) when $\rho = 1$. By substituting the formula for the two-dimensional normal distribution, which in this case has the form of

$$p(x,\ y) = p(x)\,\delta[(y-\bar{y})-(x-\bar{x})],$$

in the expression for the characteristic function, we get

$$\theta = e^{iu_1\bar{x}+iu_2\bar{y}}\,G(u_1+u_2), \tag{1.33}$$

where

$$G(u_1+u_2) = \int_{-\infty}^{\infty} p(x-\bar{x})\,e^{i(x-\bar{x})(u_1+u_2)}\,d(x-\bar{x}); \tag{1.34}$$

b) when $\rho = -1$, by using the analogous formula for the two-dimensional distribution, we obtain the following expression for the characteristic function

$$\theta = G_1(u_1-u_2)\,e^{iu_1\bar{x}+iu_2\bar{y}}. \tag{1.35}$$

From relationships (1.32) and (1.33) we get:

$$G(u_1,\ u_2) = i(u_1\bar{x}+u_2\bar{y})+G_1(u), \tag{1.36}$$

where

$$G_1(u) = \ln G(u_1+u_2)+u_1u_2.$$

Similarly on the basis of (1.35) and (1.32) we get:

$$G(u_1,\ u_2) = i(u_1x+u_2y)+G_1(u), \tag{1.37}$$

where

$$G_1(u) = \ln G(u_1-u_2)-u_1u_2.$$

Expressions for function G which are found on the basis of conditions a) and b) should be equal. The relationships obtained above are valid for arbitrary random processes x(t), y(t). Thus, u_1, u_2 can be regarded as independent variables. Under these

conditions the equality of expressions (1.36) and (1.37) is only possible when

$$G_1(u) = \mathrm{const} = Q.$$

The expression for the characteristic function in this case

$$\theta(u_1, u_2) = \exp\{-u_1u_2K(\tau) + i(u_1x + u_2y) + Q\} \qquad (1.38)$$

corresponds to the two-dimensional gaussian distribution of [24]. Constant Q can be considered equal to zero when the appropriate normalization standards are fulfilled.

Thus, the solution and analysis of expression (1.31) have shown that the basic relationship of (1.25) is fulfilled under the condition that random processes x(t) and y(t) are gaussian processes.

The dependence between correlation functions of gaussian random processes before and after nonlinear transformations are found on the basis of (1.25). Considering the averaging of the set in the right part of (1.25) we get

$$\frac{\partial^k \psi(\tau)}{\partial K^k(\tau)} \quad \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} f_1^{(k)}(x)\, f_2^{(k)}(y)\, p(x, y)\, dy. \qquad (1.39)$$

where p(x, y) is the two-dimensional normal probability density.

Nonlinear transformations and processes which can be performed before correlation lead to a change in the form of the correlation function. For example, in the case of a bilateral restriction, where

$$f_1(x) = f_2(x) = \begin{cases} 1 \text{ при } x > 0, \\ -1 \text{ при } x < 0. \end{cases} \qquad (1.40)$$

[при = when]

the relationship between the correlation functions before and after nonlinear transformation will have the form of

$$\psi(\tau)=\frac{2}{\pi}\arcsin|K(\tau)|. \tag{1.41}$$

Measurement error (the difference $\psi(\tau) - K(\tau)$) is described by the curves shown in Fig. 1.12. Correlators in which there is a transformation of the studied random processes according to (1.41) are called correlators with polarity coincidence. The value of the output effect of the correlator is determined by the probability of coincidence (or lack of it) in the polarity of the studied processes. It is technically easier to produce correlators operating on the indicated principle than correlators with ideal multiplication (see § 2.1).

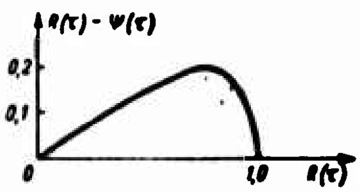

Fig. 1.12. Measurement error curve for correlation function with respect to polarity coincidence.

It is also possible to use the polarity coincidence method to create correlators with a nongaussian distribution of the studied random processes, although the difference between $\psi(\tau)$ and $K(\tau)$ in this case may be considerable [65].

In correlation devices with a nonlinear transformation in only one channel when certain conditions are met for the distribution function of the probability density of the processes and the transformation characteristic [48, 49], correlation functions $\psi(\tau)$ and $K(\tau)$ differ only in a constant factor, i.e., in scale. The indicated result is valid for more than the gaussian distribution.

This analysis does not enable us to judge the deterioration in the noise characteristics of correlation devices which contain nonlinear transformations.

SECOND CHAPTER

CORRELATORS

2.1. DESIGN PRINCIPLE OF CORRELATORS

According to the algorithm of (1.2) or (1.[illegible]) the correlator is a device consisting of a multiplication system for input signals and an integrator which integrates the results of the multiplication. The presence of a statistical connection between the input processes results in a d-c component at the multiplier output. The d-c component passes through the integrator which decreases response fluctuation at the output.

In correlator design one frequently encounters serious difficulty in achieving a sufficiently rapid multiplication device. Current analog multiplication methods can be broken down into two groups - direct and indirect. The first group contains systems which employ physical effects proportional to the product of the two measured quantities - dual control of the anode current by the electron tube, the Hall effect, etc.

Indirect multiplication systems perform such multiplication operations as

$$xy = \frac{1}{4}\left[(x+y)^2 - (x-y)^2\right].$$

The operations of adding and subtracting can be performed by transformers, electronic systems, resistors, etc. The initial section of the anode-grid characteristic of the tube triode or a section of the volt-ampere characteristic of the semiconductor diode, special electron tubes with a parabolic characteristic, Diode circuits, etc., may be used as the square-law generator.

We must mention a group of devices which belong to the direct multiplication systems - linear four-pole networks with variable parameters. The work of these devices is based primarily on the principle of controlling the transmission coefficient of the four-pole network. The voltage of one factor is supplied to the input of this four-pole, and the transmission coefficient is varied in proportion to the voltage of the other factor. These systems include different types of modulators.

The methods discussed above make it possible to multiply the studied processes continually by simple methods which provide a high degree of accuracy.

The design of correlators can be facilitated if a deviation from the algorithm of (1.8) and (1.9) is permitted and if in place of multiplying the studied processes the functions of these processes are multiplied. There exist various methods of designing such correlators, including time- and level-digitization of the studied processes.

Let us examine a correlator whose action can be described in diagrams (Fig. 2.1). Curves a_1 and b_1 represent realizations of the studied random processes. Graphs a_2 and b_2 show the two periodic pulse trains with a repetition period of ΔT, obtained from a single source of harmonic oscillations. We will assume that intervals ΔT are considerably smaller than the correlation times of the studied processes. The second pulse train is time-shifted by means of the delay device relative to the first by delay

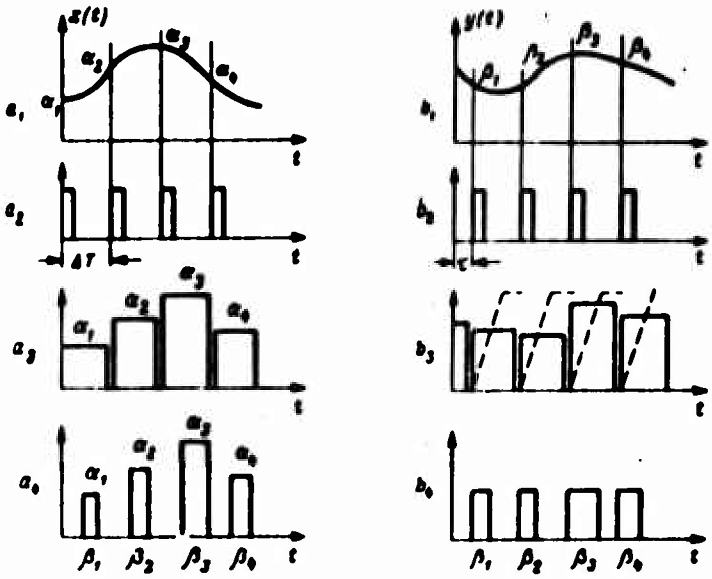

Fig. 2.1. Conversion of studied processes in correlator.

time τ. Pulse train a_2 gives us the values of the first random process in discrete moments in time. Wide rectangular pulses a_3 are formed such that their amplitudes are proportional to references α_1, α_2, ..., α_n, while their length is constant and equal approximately to ΔT. When $\Delta T < \tau_k$ through transformation continuous random process a_1 is transformed into process a_3 with a discrete change in the envelope. Similarly the sequence of wide pulses b_3 is formed, whose amplitudes are proportional to references β_1, β_2, ..., β_n. By means of sawtooth voltage the amplitude-modulated pulse train of the second random process b_3 is transformed into length-modulated pulses b_4. The width of the pulses obtained is proportional to β_1, ..., β_n. By means of the coincidence system pulse currents (a_3, b_4) are multiplied, forming a new pulse current, whose amplitude is proportional to references α_1, ..., α_n, while the width of the pulses is proportional to references β_1, ..., β_n. Integrating a pulse train for a large number of references gives us a quantity which is proportional to the value of the correlation function.

The operations performed in the correlator may be briefly represented as follows. For each of the processes respective

values of $a(t_i)$ and $b(t_i)$ are assigned, which are then multiplied and added over time $N\Delta T$, i.e., the sum

$$R_1(\tau) \approx \frac{1}{N}\sum_{i=1}^{N} a(t_i)\, b(t_i+\tau). \tag{2.1}$$

is formed, which represents an approximate expression for the correlation integral (1.9).

Certain correlation functions may be similarly performed, if we take our references at time intervals of $\Delta T > \tau_k$. In this case for each of the random processes we can assume that subsequent references are not correlated with one another. Each of the processes is broken down into N segments, each of which has a length of ΔT. The segments of the random process may be regarded as its separate occurrences. In each of these realizations we take one reference, which for the first process corresponds to the beginning, while a time shift is introduced for the second process. We can assume that the sum of (2.1) in this case expresses the integral of (1.6) for the correlation function when the set is averaged. For the studied random processes the ergodic property is valid.

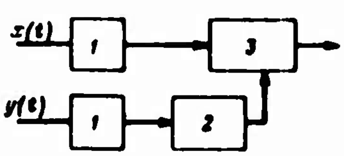

Fig. 2.2. Block diagram of digital correlator: 1 - analog-digital converter; 2 - delay device; 3 - arithmetic unit.

Computer methods make it possible to achieve the correlator design method discussed above by other means. In the correlator shown in Fig. 2.2 the values of the studied processes which are assigned at discrete moments of time are converted into a digital form and enter the arithmetic unit, which performs operations equivalent to multiplication and integration. One of the studied processes can also be delayed by means of digital devices, for example, shift registers.

In designing correlators greater deviations from the algorithm (1.8) can be tolerated than in the case discussed above; for example, information concerning only the moments when the processes pass through zero would be considered and information concerning the envelopes discarded. An example of such correlators (see § 1.4) would be sign correlators (or polarity-coincidence correlators). The simplified functional system of a sign correlator is similar to that shown in Fig. 2.2. The purpose of the blocks in this case would be different: 1 - limiter, 2 - delay unit, 3 - logic multiplication block. Voltages corresponding to the studied random processes are amplitude-limited. In this case only information concerning a sign change in the random process is preserved. Multiplication is done by means of the coincidence circuit, which can be obtained with relatively little difficulty. Based on the intensity of the regular component of the output voltage the degree of correlation between random processes can be determined.

In studying Gaussian processes by means of a sign correlator errors develop in measuring the correlation function, some of which are discussed in § 1.4 (see Fig. 1.12). In analyzing processes with a nongaussian distribution, the connection between correlation functions obtained by algorithms (1.8) and (.19) and those obtained by means of a sign correlator becomes more complex [65].

A method is known for measuring correlation functions in which results are obtained which are close to the ideal multiplication method. This is achieved by introducing special auxiliary random processes, and results in complication of the system [28].

In the case of cross-correlation reception of weak signals in the presence of noise, good results can be obtained from a correlator design [28, 59] in which only one stored signal is subject to limitation. The random process which is received (a mixture of signal and noise) is not subject to transformation. In

this case the output voltage of the correlator is proportional to the value of the correlation function of the studied processes. Correlators of this type are sometimes called relay correlators [2].

Random process transformation corresponding to the action principle of sign and relay correlators, cannot be performed accurately in practice due, for example, to the presence of non-linear and inertial elements in the systems. The actual functional transformations may be more complex than one might assume in studying the action principle of the correlators.

The method of designing correlators is based on representing the correlation function in a series in a complete system of functions [2, 52]. The function which exists in the interval (0, ∞) and which satisfies the conditions

$$\lim_{u \to \infty} f(u) \to 0; \qquad \int_0^\infty e^{-\frac{u}{2}} x^{\frac{\alpha}{2} - \frac{1}{4}} |f(u)| \, du < M,$$

where M is a finite number, can be represented in the form of a series, of Laguerre polynomials, for example:

$$f(u) = \sum_{n=0}^{\infty} C_n L_n^{\alpha}(u), \qquad (2.2)$$

where $\alpha > -1$,

$$C_n = \frac{n!}{\Gamma(n+\alpha+1)} \int_0^\infty f(u) \, e^{-u} u^{\alpha} L_n^{\alpha}(u) \, du,$$

while $\Gamma(z)$ is the gamma-function, which is determined by formula

$$\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} \, dt.$$

Of interest also are simple Laguerre polynomials corresponding to $\alpha = 0$.

Series (2.2) converges at all points where function f(u) is continuous. At the break point the series converges to $\frac{1}{2}[f(u+0) + f(u-0)]$. The Laguerre polynomials are determined by the formula

$$L_n^{\alpha}(u) = e^{u}\frac{u^{-\alpha}}{n!}\frac{d^n}{du^n}(e^{-u}u^{n+\alpha}), \quad n = 0, 1, 2 \ldots \qquad (2.3)$$

They are orthogonal on the interval $(0, \infty)$ with a weight of $u^{\alpha}e^{-u}$.

When $\tau > 0$ the correlation function satisfies the above mentioned conditions of the series representation.

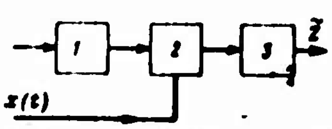

Fig. 2.3. Shaping system of voltage proportional to the coefficient of series (2.2).

The task of the correlator is to determine the coefficients of expansion C_n from the studied occurrence of the signal, and then to shape the unknown correlation function from the coefficients found. The coefficients can be determined by means of the system shown in Fig. 2.3. Voltage at the output of filter 1, which has a pulse characteristic of $h_n(t)$, is equal to

$$y(t) = \int_0^{\infty} x(t-\tau)\,h_n(\tau)\,d\tau.$$

After multiplication and averaging, assuming that the ergodic property is valid, we get the quantity

$$\overline{z_n(t)} = \int_0^{\infty} R(\tau)\,h_n(\tau)\,d\tau,$$

which according to the condition that $h_n(\tau) = \beta e^{-\beta\tau}L_n(\beta\tau)$ is equal to $C_n \times \frac{\Gamma(n+1)}{n!}$. Here β is a constant, which is determined from the

conditions of obtaining the least error with a finite number of filters.

It is technically possible to produce filters with this characteristic. By means of simultaneously working devices such as that shown in Fig. 2.3 it is possible to obtain C_n coefficient corresponding to different n values.

The correlation function can be found by means of a device such as that shown in Fig. 2.4. It also contains filters, just as shown in Fig. 2.3. Voltage in the form of $\beta e^{-\beta\tau} L_n(\beta\tau)$ is formed by stimulating the filters with short pulses (δ-function). The output voltages of the filters are multiplied by corresponding C_n coefficients, obtained by means of the system shown in Fig. 2.3, and are added. The second multiplier is needed to eliminate the weight factor in the sum which is formed. As a result a voltage which represents the correlation function according to formula (2.2), where $u = \beta\tau$, will be obtained at the output. The advantage of correlators of this type is time reduction in analyzing correlation characteristics of random processes.

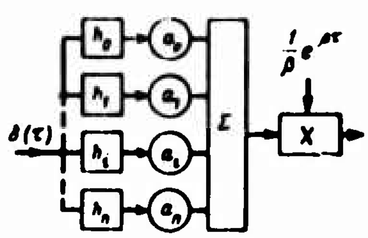

Fig. 2.4. System for shaping voltage proportional to correlation functions.

Indirect methods are known for measuring the normalized correlation function, for example, by analyzing the level curves for two-dimensional probability distribution density. For gaussian random processes curves $p(x, y)$ = const have elliptical shapes, whose axes are turned in relation to the coordinate axes of random variables x and y. (For other distribution laws the shape of the region will be different.) The ratio of a small ellipse axis to a large axis under identical process dispersions depends on the correlation coefficient: $\rho=(1-a)/(1+a)$.

where α is the ratio of the axes of the ellipse. The position of the ellipse on the plane determines the sign of the correlation coefficient. The sign "+" corresponds to the position of the large axis in quandrants I and III; the sign "-" corresponds to quadrants II and IV.

This method can be accomplished if a cathode-ray tube is used. The studied processes (in the form of voltages) arrive at plates X and Y. Because of the nonlinear properties of the luminophor on the screen, one can observe the outlined boundary of the glowing region, which consists of one of the curves of equal brightness, i.e., function $p(x, y) = \text{const}$. For gaussian processes the correlation coefficient is determined by the formula presented above.

This method is of secondary importance, since it requires additional calculations and has a low degree of accuracy. Its advantage lies in its simplicity.

2.2. CHARACTERISTICS OF IDEAL DIFFERENCE FREQUENCY CORRELATOR

As an ideal correlator we mean a correlator containing ideal multiplication according to algorithm (1.8) or (.19). Averaging is done in finite time. An interesting case is one where the spectral densities of the studied processes are frequency-mixed.

In actual multiplying devices there occurs not only multiplication, but also signal detection. Detection leads to a d-c voltage component at the output of the multiplier which is unrelated to the presence of correlation between input processes. If the input signals are sufficiently powerful, the d-c component of the detector effect can considerably increase the d-c component causing cross-correlation between weakly correlated input processes. If the processes at the correlator input are strictly stationary, then

the d-c component of the detector effect can be excluded by means of compensation voltage. However, actual radioengineering units always have nonstationarity. For example, the passage of a weak signal through an amplifier is accompanied by power modulation of this signal because of unavoidable fluctuations in the amplification coefficient. Thus, the above compensation method is not effective in an overwhelming majority of cases. The situation in this case is analogous to zero drift in amplifiers with d-c voltage.

To eliminate the influence of the detector effect mutual displacement in the spectra of input processes can be introduced. In most technical applications this does not result in serious difficulty and even makes it possible to create correlators for studying processes with a wide energy spectrum of hundreds of MHz. Correlators which function under a shift in the spectra of one signal relative to another we will call difference frequency correlators.

Let us analyze the work of an ideal difference frequency correlator under the following assumptions:

1. Two correlation processes are transmitted to the correlator: the signal $u_c(t)$ and the reference voltage $u_r(t)$, which differ in their strength and time delay by $\tau_з$. The signal enters with extraneous noise.

2. The signal and the extraneous noise $u_ш(t)$, which is not correlated with the signal and the reference voltage, enters one multiplier input additively; the reference voltage enters a second input. At the multiplier output there is an averaging filter whose band pass is much narrower than the energy spectra of the input vibrations.

3. The spectrum of the reference voltage is shifted by frequency $\omega_0 = 2\pi f_0$ relative to the signal spectrum. This

shift might be achieved by the single-band modulation methods, for example.

4. The studied processes represent occurrences of narrow-band gaussian noise with energy spectra which are symmetrical in relation to the central frequencies. We will describe them by the random functions of (1.16).

5. The average values of the processes are equal to zero, while the correlation function under the assumptions which we have made are determined by expressions such as (1.19). Thus, for the signal we have

$$R_c(\tau) = \sigma_c^2 r_c(\tau) \cos \omega_c \tau, \qquad (2.4)$$

where σ_c^2, ω_c, $r_c(\tau)$ is dispersion, the average frequency, and the envelope of the autocorrelation function of the signal.

The autocorrelation functions of reference voltage and extraneous noise are expressed by similar relationships of the type of (2.4), although the subscript "c" is replaced by "r" and "ш" respectively.

The final goal of the analysis is to determine signal/noise ratios at the correlator output. The studied problem has a direct relationship, for example, to the analysis of the noise characteristics in a radar station operating under a noise signal.

Let us examine the correlation properties of oscillations entering the multiplier input. The cross-correlation function of the signal and the base voltage has a nonstationary nature. To prove this let us first say that the correlated reference voltage and signal have the same central frequencies, i.e., $\omega_c = \omega_r = \omega_1$, while $\tau_з = 0$. Then, after writing the reference voltage and signal in the form of random time functions of the type (1.16), we get

$$
\begin{aligned}
&\overline{u_c(t_2)\,u_r(t_1)} = \\
&= \overline{E_r(t_1)\cos[\omega_1 t_1 + \varphi_r(t_1)]\,E_c(t_2)\cos[\omega_1 t_2 + \varphi_c(t_2)]} = \\
&= \frac{1}{2}\{\overline{E_r(t_1)E_c(t_2)\cos[\varphi_c(t_2) - \varphi_r(t_1)]}\cos\omega_1(t_2 - t_1) - \\
&- \overline{E_r(t_1)E_c(t_2)\sin[\varphi_c(t_2) - \varphi_r(t_1)]}\sin\omega_1(t_2 - t_1) + \\
&+ \overline{E_r(t_1)E_c(t_2)\cos[\varphi_c(t_2) + \varphi_r(t_1)]}\cos\omega_1(t_2 + t_1) - \\
&- \overline{E_r(t_1)E_c(t_2)\sin[\varphi_c(t_2) + \varphi_r(t_1)]}\sin\omega_1(t_2 + t_1)\}.
\end{aligned} \tag{2.4a}
$$

In this relationship the last two terms are equal to zero, since when $\omega_c = \omega_r = \omega_1$ the reference voltage and the signal have a stationary relationship and, consequently, their cross-correlation function should depend solely on the difference $t_2 - t_1$. Thus,

$$
\begin{aligned}
&\overline{E_r(t_1)E_c(t_2)\cos[\varphi_c(t_2) + \varphi_r(t_1)]} = 0, \\
&\overline{E_r(t_1)E_c(t_2)\sin[\varphi_c(t_2) + \varphi_r(t_1)]} = 0.
\end{aligned} \tag{2.4b}
$$

Consequently, in expression (2.4a) there remain two terms, which can be reduced to the form of

$$
\begin{aligned}
u_r(t_1)u_c(t_2) = \sigma_r\sigma_c r_B(t_2 - t_1)\times\cos[\omega_1(t_2 - t_1) + \\
+ \zeta_B(t_2 - t_1)],
\end{aligned} \tag{2.5a}
$$

where $r_B(t_2 - t_1)$, $\zeta_B(t_2 - t_1)$ is the envelope and the phase of the normalized cross-correlation function of random processes $u_r(t)$ and $u_c(t)$. If $\omega_r \neq \omega_c$, then the cross-correlation function of the signal and the reference voltage will have the form of

$$
\begin{aligned}
&R_{rc}(t_1, t_2) = \\
&= \overline{E_r(t_1)\cos[\omega_r t_1 + \varphi_r(t_1)]\times E_c(t_2)\cos[\omega_c t_2 + \varphi_c(t_2)]} = \\
&= \frac{1}{2}\{\overline{E_r(t_1)E_c(t_2)\cos[\varphi_c(t_2) - \varphi_r(t_1)]}\cos(\omega_c t_2 - \omega_r t_1) - \\
&\quad \overline{E_r(t_1)E_c(t_2)\sin[\varphi_c(t_2) - \varphi_r(t_1)]}\sin(\omega_c t_2 - \omega_r t_1) + \\
&+ \overline{E_r(t_1)E_c(t_2)\cos[\varphi_r(t_1) + \varphi_c(t_2)]}\cos(\omega_r t_1 + \omega_c t_2) - \\
&\quad \overline{E_r(t_1)E_c(t_2)\sin[\varphi_r(t_1) + \varphi_c(t_2)]}\sin(\omega_r t_1 + \omega_c t_2)\}.
\end{aligned}
$$

According to the equality of (2.4b) the mathematical expectations in the last two terms are equal to zero. Finally we can write

$$R_{rc}(t_1, t_2) = \sigma_r \sigma_c r_в(t_2 - t_1) \times \\ \times \cos[\omega_c t_2 - \omega_r t_1 + \zeta_в(t_2 - t_1)]. \qquad (2.5b)$$

Thus, in the case analyzed the time dependence of the cross-correlation function $R_{rc}(t_1, t_2)$ has a periodic nature.

Let us study the characteristics of the random process at the multiplier output. The regular response component of the multiplier will equal

$$\overline{u_н(t, \tau_3)} = \overline{u_r(t)[u_c(t + \tau_3) + u_ш(t)]}.$$

If we consider relationship (2.5b), as well as the fact that the reference voltage and extraneous noise are independent, then we get

$$\overline{u_н(t, \tau_3)} = \sigma_c \sigma_r r_в(\tau_3) \times \\ \times \cos[\omega_0 t - \omega_c \tau_3 + \zeta_в(\tau_3)],$$

where $\omega_0 = \omega_r - \omega_c$ is the intermediate (difference) frequency. Thus, under the indicated conditions the regular component of the output voltage of the multiplier indicating the degree of correlation between the signal and the reference voltage is isolated at difference frequency ω_0. The amplitude of harmonic voltage is proportional to the envelope of the cross-correlation function of random processes $u_c(t)$ and $u_r(t)$.

Let us examine noise at the correlator output. The correlation function of the process at the multiplier output will have the form of

$$K_н(t, \tau) = \overline{u_н(t, \tau_3)\, u_н(t + \tau, \tau_3)}.$$

If we remove the parentheses and assume that for normal random processes with zero average values the following relationship [27, p. 447]

$$\overline{x_1 x_2 x_3 x_4} = \overline{x_1 x_2}\,\overline{x_3 x_4} + \overline{x_1 x_3}\,\overline{x_2 x_4} + \overline{x_1 x_4}\,\overline{x_2 x_3},$$

we get

$$\begin{aligned} K_u(t,\tau) &= R_{\text{г}}(\tau) R_{\text{c}}(\tau) + R_{\text{г}}(\tau) R_{\text{ш}}(\tau) + \\ &+ \overline{u_{\text{г}}(t) u_{\text{c}}(t+\tau_0)}\;\overline{u_{\text{г}}(t+\tau) u_{\text{c}}(t+\tau+\tau_0)} + \\ &+ \overline{u_{\text{г}}(t) u_{\text{c}}(t+\tau+\tau_0)}\;\overline{u_{\text{г}}(t+\tau) u_{\text{c}}(t+\tau_0)}. \end{aligned}$$

Considering the fact that the autocorrelation functions of the studied processes are determined by relationships in the form of (1.1[illegible]), while the cross-correlation function of the signal and the reference voltage are determined by a relationship in the form of (1.[illegible]), we get the following expression for the time-averaged correlation function of the output voltage of the multiplier:

$$\begin{aligned} \langle K_u(t,\tau)\rangle &= \sigma_{\text{c}}^2 \sigma_{\text{г}}^2 r_{\text{г}}(\tau) r_{\text{c}}(\tau) \cos\omega_{\text{г}}\tau \cos\omega_{\text{c}}\tau + \\ &+ \sigma_{\text{г}}^2 \sigma_{\text{ш}}^2 r_{\text{г}}(\tau) r_{\text{ш}}(\tau) \cos\omega_{\text{г}}\tau \cos\omega_{\text{ш}}\tau + \frac{1}{2}\sigma_{\text{c}}^2 \sigma_{\text{г}}^2 r_{\text{в}}(\tau + \\ &+ \tau_0) r_{\text{в}}(\tau_0 - \tau) \cos\left[(\omega_{\text{г}} + \omega_{\text{c}})\tau + \zeta_{\text{в}}(\tau + \tau_0) - \right. \\ &\left. - \zeta_{\text{в}}(\tau_0 - \tau)\right] + \frac{1}{2}\sigma_{\text{г}}^2 \sigma_{\text{c}}^2 r_{\text{в}}^2(\tau_0) \cos\omega_0 \tau_0, \end{aligned}$$

where < > denotes time averaging.

To find the energy spectrum of the response of the multiplier we must calculate the Fourier transform from the time-averaged correlation function. From the expression for this function it follows that the energy spectrum of the response is concentrated near frequencies $\omega_{\text{г}} \pm \omega_{\text{c}}$, $\omega_{\text{г}} \pm \omega_{\text{ш}}$. Generally the intermediate frequency ω_0 which is selected is much smaller than the average signal frequencies, the reference voltage, and extraneous noise, since amplification is easier at relatively low frequencies. At

the same time, in order to eliminate the harmful effect of detector type noise, which occurs in actual correlators, we must select an intermediate frequency which is considerably wider than the energy spectrum of any of the input oscillations.

Let us assume that these conditions are fulfilled and, consequently, only noise whose energy spectra are concentrated near difference frequencies $\omega_{\Gamma} - \omega_{C}$ and $\omega_{\Gamma} - \omega_{Ш}$ enter the band pass of the averaging filter at the multiplier output. The time-averaged correlation function of the response of the multiplier, which describes the output voltage in the intermediate frequency range, is equal to

$$R_{в}^{пч}(\tau) = \langle K_{в}(t, \tau)_{пч} \rangle = \frac{1}{2} \sigma_{c}^{2} \sigma_{г}^{2} r_{в}^{2}(\tau_{з}) \cos \omega_{в}\tau +$$
$$+ \sigma_{c}^{2} \sigma_{г}^{2} r_{c}(\tau) r_{г}(\tau) \cos \omega_{в}\tau + \sigma_{ш}^{2} \sigma_{г}^{2} r_{ш}(\tau) r_{г}(\tau) \cos (\omega_{г} - \omega_{ш})\tau.$$

The first two terms of this equation reflect the correlation function of a signal which has been transformed by the reference voltage. In this case only the first term corresponds to the correlation function of the regular component. The second term determines the correlation function of noise which develops as the result of the fluctuating nature of the studied processes. For the sake of simplicity we will call noise of this type self-noise. (In analyzing extremely weak input signals it may be necessary to consider the internal noise of the correlator, i.e., noise caused by losses in the elements of the system and by the discrete nature of the current carriers. We consider internal noise in the same manner as extraneous noise.) From the obtained expressions it follows that the energy spectrum of these noises does not depend on time delay $\tau_{з}$ of the signal relative to the reference voltage (this has also been confirmed experimentally). Let us find the effectiveness of an ideal correlator for a particular case, where

$$r_{г}(\tau) = r_{c}(\tau) = r_{в}(\tau) = r(\tau),$$
$$\tau_{з} = 0, \ \omega_{ш} = \omega_{c}, \ \zeta_{в}(\tau) \equiv 0. \qquad (2.5c)$$

In fulfilling the conditions of (2.5c) the time-averaged correlation function of voltage at the multiplier output for the intermediate frequency range is equal to

$$R_{\text{ш}}^{\text{пч}}(\tau) = \frac{1}{2}\sigma_c^2\sigma_r^2\cos\omega_0\tau\left[1 + r^2(\tau) + \frac{\sigma_{\text{ш}}^2}{\sigma_c^2}r(\tau)\cdot r_{\text{ш}}(\tau)\right].$$

The corresponding energy range is found by means of the Wiener-Khinchin transform:

$$S_{\text{ш}}(f) = \frac{1}{4}\sigma_c^2\sigma_r^2\left\{[\delta(f-f_0) + \delta(f+f_0)] + [{}_2S_c(f-f_0) + \right.$$
$$\left. + {}_2S_c(f+f_0)] + \frac{\sigma_{\text{ш}}^2}{\sigma_c^2}[{}_2S_{r\text{ш}}(f-f_0) + {}_2S_{r\text{ш}}(f+f_0)]\right\}. \qquad (2.6)$$

where $\delta(f)$ is the delta-Dirac function;

$${}_2S_c(f) = \int_{-\infty}^{\infty} r^2(\tau)e^{-i2\pi f\tau}d\tau; \quad {}_2S_{r\text{ш}}(f) = \int_{-\infty}^{\infty} r(\tau)r_{\text{ш}}(\tau)e^{-i2\pi f\tau}d\tau.$$

Note that in this determination the energy spectrum is assigned for the entire axis of the frequencies $(-\infty, \infty)$.

In (2.6) the first term in brackets determines the spectral denstiy of the power of the regular output voltage component, which appears in the presence of a correlation connection between the reference voltage and the signal; the second term describes self-noise caused by the fluctuating nature of the signal; the third extraneous noise transformed by the reference voltage.

In examining the output effect of the correlator the question arises: does the self-noise of the signal belong to the signal component of the output voltage or to the noise component? This apparently depends on the type of unit which follows the averaging filter. If it is a threshold system with a two-alternative test for the hypothesis of the presence or absence of the signal, then

self-noise which appears only in the presence of a signal and thus increases the probability of correct detection should belong to the signal component. In many cases self-noise impairs accuracy in parameter measurements, in which case it must be referred to the noise component of the output effect. In our calculations we will have the second case in mind.

Let the narrow-band averaging filter which stands behind the multiplier have an amplitude-frequency characteristic H(f) with a maximum on frequency f_0. On the basis of expression (2.6) we find the power of the harmonic voltage at the output of the averaging filter

$$P_c = \frac{1}{4}\sigma_c^2\sigma_r^2 \int_{-\infty}^{\infty} [\delta(f - f_0) + \delta(f + f_0)] H^2(f)\, df =$$

$$= 0{,}5\sigma_c^2\sigma_r^2 H^2(f_0).$$

In deriving this relationship the property of the delta-function is considered, i.e.,

$$\int_{-\infty}^{\infty} \delta(f - f_0) H^2(f)\, df = H^2(f_0).$$

as well as the parity property of the amplitude-frequency characteristic of the averaging filter H(-f) = H(f). The power of self-noise at the output of the averaging filter equals

$$P_{ш}^{сш} = \frac{\sigma_c^2\sigma_r^2}{4} \int_{-\infty}^{\infty} [{}_2S_c(f - f_0) + {}_2S_c(f + f_0)] H^2(f)\, df.$$

The band of the averaging filter is assumed to be considerably smaller than the energy spectrum band of the processes at the correlator output. Thus, the integrand represents the product of slowly changing frequency functions ${}_2S_c(f - f_0)$ or ${}_2S_c(f + f_0)$ times rapidly changing $H^2(f)$ with a maximum on frequency f_0. Under

these conditions the value of the integral is practically determined by the values of the slowly changing function on frequencies f_0 and $-f_0$. If we consider the parity property of the energy spectrum, we get

$$P_{\text{ш}}^{\text{сб}} = \frac{\sigma_c^2 \sigma_r^2}{4} [{}_2S_c(2f_0) + {}_2S_c(0)] \int_{-\infty}^{\infty} H^2(f)\,df.$$

With similar reasoning we find the expression for the power of extraneous noise at the averaging filter output:

$$P_{\text{ш}}^{\text{ст}} = \frac{\sigma_r^2 \sigma_{\text{ш}}^2}{4} [{}_2S_{r\,\text{ш}}(2f_0) + {}_2S_{r\,\text{ш}}(0)] \int_{-\infty}^{\infty} H^2(f)\,df.$$

The output signal/noise ratio for an ideal difference frequency correlator is determined as the ratio of the power of the regular component to the total power of noise at the output of the averaging filter. Based on the above expressions we get:

$$q = \frac{P_c}{P_{\text{ш}}^{\text{сб}} + P_{\text{ш}}^{\text{ст}}} =$$

$$= \frac{2\sigma_c^2}{B_y \sigma_{\text{ш}}^2} \left\{ \frac{\sigma_c^2}{\sigma_{\text{ш}}^2} [{}_2S_c(0) + {}_2S_c(2f_0)] + [{}_2S_{r\,\text{ш}}(0) + {}_2S_{r\,\text{ш}}(2f_0)] \right\}^{-1}, \qquad (2.7)$$

where B_y is the effective band pass of the averaging filter, equal to

$$B_y = \frac{\int_{-\infty}^{\infty} H^2(f)\,df}{H^2(f_0)}. \qquad (2.8)$$

For example, the effective band pass of an averaging filter with a single oscillatory circuit is equal to

$$B_y = \int_{-\infty}^{\infty} \left\{ \frac{1}{1 + \left[\frac{2(f - f_0)}{\Delta f}\right]^2} + \frac{1}{1 + \left[\frac{2(f + f_0)}{\Delta f}\right]^2} \right\} df = \pi\Delta f;$$

where Δf is the band pass of the circuit at the level of -3 dB.

Relationship (2.7) reflects the known situation, according to which the signal/noise ratio at the correlator output depends only on the energy of the signal (during the accumulation time of the output averaging filter) and the spectral density of the interference. The self-noise of the signal belongs to interference.

The ideal correlator represents the technically attainable limit, and can serve as a standard in evaluating the degree of perfection in actual correlator devices.

2.3. FUNCTIONAL CORRELATORS

1. Action Principle of Functional Correlator

Let us examine devices which instead of multiplication under the integral sign form a function from the studied voltages: $f(x, y)$. The algorithm for the work of such a device is determined by the relationship

$$R(\tau)=\frac{1}{T}\int_{-T/2}^{T/2} f[x(t), y(t-\tau)]\,dt, \qquad (2.9)$$

or, if we assume that the ergodic property is valid and shift to averaging of the set, then it is determined by

$$K(\tau)=\iint_{(L)} f(x, y)\,p(x, y)\,dx\,dy. \qquad (2.10)$$

Devices which work on the basis of algorithm (2.9) will be called functional correlators in contrast to correlators containing ideal multiplication. The term "correlation detectors" is sometimes used for devices of this type in detection systems.

The most commonly used functional correlators are those for which $f(x, y) = f_1(x)f_2(y)$. An example of this might be polarity coincidence correlators (or sign correlators) for which functions f_1 and f_2 have the form:

$$f(x) = \operatorname{sgn} x = \begin{cases} 1 \text{ при } x > 0, \\ 0 \text{ при } x = 0, \\ -1 \text{ при } x < 0. \end{cases} \qquad (2.11)$$

[при = when]

Transformation (2.11) transforms the studied random processes into random processes with a constant amplitude. Information about the phase is preserved. This transformation or one similar to it can only occur for one of the studied processes; the second random process does not change [55, 59].

Some interesting facts in studying functional correlators are. a) errors in measuring the correlation function, which are the result of nonlinear transformations in the studied processes; b) deterioration of the noise characteristics in a case where, along with the studied processes, additive noise affects the input.

Particularly interesting in designing functional correlators are dependences $f(x, y)$ for which:

1) function $K(\tau)$ differs little from the correlation function found under ideal multiplication of $K_и(\tau)$. The criterion might, for example, be the maximal value of the relative difference $\frac{1}{K_и(\tau)} |K_и(\tau) - K(\tau)|$ or its square;

2) the noise properties of the functional correlator do not decline significantly as compared to the ideal correlator. The decline in the signal/noise ratio at the output of the functional correlator can serve as the criterion;

3) the functional correlator is technically simpler and its dependability higher than a correlator with ideal.multiplication.

The first condition can be written analytically. It leads to the integral equation

$$\int_{L} f(x, y)p(x, y)\, dx dy = K_{u}(\tau) + \varkappa(\tau), \qquad (2.12)$$

where function $\kappa(\tau)$ describes permissible error in measuring the correlation function. By solving equation (2.12) it is theoretically possible to find the forms of functional dependences f(x, y) which will give the necessary accuracy in the work of the functional correlator (if such a solution exists). The next step should be to study the noise properties and to examine the conditions of technical realization.

In the method which has been formulated for analyzing functional correlators there are great mathematical difficulties associated with solving the uniform integral equation (2.12) [31].

Thus, in analyzing functional correlators it is best to use the simplest approach, which is as follows:

1) Examined are functions f(x, y), corresponding to the characteristics of nonlinear elements which have broad technical application;

2) for the selected characteristics f(x, y) the correlation function is found according to formula (2.10) and possible measurement error is estimated;

3) the noise properties of functional correlators are studied and compared with the noise properties an ideal correlator.

The simplified analysis method will be used in the fourth and fifth chapters in studying functional correlators.

2. Simplest Functional Correlator Systems

Various construction methods and systems are known for correlators which contain electron tubes and semiconductor elements [2, 19, 23, 28]. Let us dwell briefly on simple functional correlator systems whose work algorithm is determined by relationship (2.9).

In the low frequency and radio frequency ranges a correlator system based on a balanced ring mixer gives good results [56, 59]. One design variation of such a system is shown in Fig. 2.5. The mixer can work in a nonlinear transformer regime as well as in a linear system with variable parameters. The first working mode occurs in the case where the intensities of the studied voltages in the first approximation are the same. In the second working mode the intensity of one of the voltages is considerably greater than that of the other. A bridge balance system is used to prevent the studied random voltages from passing directly to the output. Direct passage creates additional error in measuring correlation dependences. In this case, where the averaging filter at the correlator output does not transmit the spectral components of the studied signals (for example, in difference frequency correlators), the system may not be balanced and may be used to switch just one diode. This mode may be used, for example, in microwave correlators.

The reference voltage which switches diodes $Д_1$-$Д_4$ enters input 1. When the voltage on the cathode of phase-inversion stage $Л_1$ exceeds the voltage on the anode, the diode pair $Д_1$, $Д_2$ is triggered; otherwise diodes $Д_3$, $Д_4$ are triggered. In this case the signal transmitted to input 2 enters the grids of tubes $Л_3$ and $Л_4$. The triodes of $Л_3$ and $Л_4$ convert the voltage which is taken

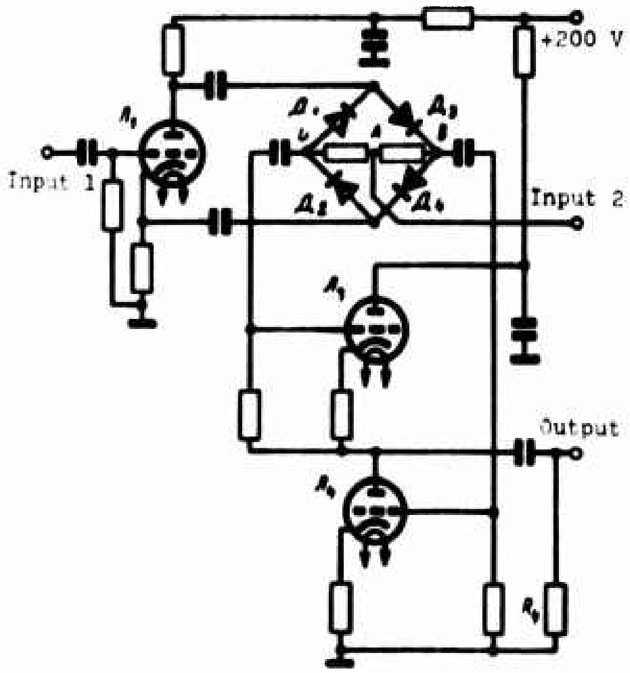

Fig. 2.5. Simplified scheme of functional correlator.

off points Б and В of the bridge, which is symmetrical with the ground, into an asymmetrical voltage, separated at resistance R_4.

Diodes $Д_1$-$Д_4$ work in an "on-off" mode. This is confirmed by the characteristic (Fig. 2.6) indicating the dependence of voltage at the system output on the effective value of the reference voltage at input 1 when the amplitude of the voltage at input 2 is constant. If reference voltage exceeds quantity u_0, the system works in a "key" mode, i.e., the conductivity of diodes in the first approximation acquire only two discrete values corresponding to the direct and reverse voltages applied to the diode. The effective value selected for the reference random voltage which is supplied to input 1 is considerably greater than u_0. The average frequencies of the studied voltage spectra should be different. In this case, when there is correlation between the signal and reference voltage at the output, a difference-frequency voltage develops, whose amplitude is proportional to the envelope of the correlation coefficient. The output voltage in Fig. 2.5 is supplied to a narrow-band amplifier, which also separates the harmonic component of the difference frequency At the output, in addition to the harmonic oscillation, there will be a noise voltage, which is caused by the fluctuating nature of changes in the envelope of the input signal and by the finite averaging time. When $R(\tau) \to 0$ at the output only noise voltage is present.

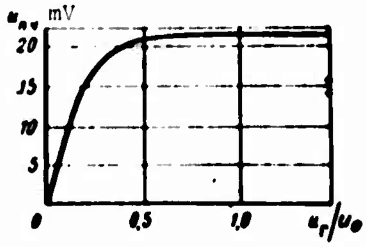

Fig. 2.6. Amplitude of output signal as a function of amplitude of reference voltage.

The studied system makes it possible to measure correlation dependences for gaussian random processes with an error close to that of a correlator with ideal multiplication.

In a case where the input and reference voltages are phase-manipulated signals, the following voltage is formed at the correlator output

$$u(t) = Ax_1(t)x_2(t) \cdot \cos(\omega_1 t + \delta_1) \cdot \cos(\omega_2 t + \delta_2),$$

where A = const, $x_1(t)$, $x_2(t)$ are modulating pseudorandom video signals; δ_1 and δ_2 are the initial phases.

Output voltage is averaged by means of a filter. Each of the modulating video signals acquires a value of +1 or -1. Table 2.1 shows the possible values of modulating video signals and the value of the output voltage (product) corresponding to them. If the modulating voltages are coherent, i.e., if the time position of their symbols coincides and if the length is the same, then output voltage will be harmonious with constant amplitude. In the case of a time shift in the modulating voltages by more than one symbol, the pseudorandom nature of output voltage modulation is preserved. In the case of a time shift by less than one symbol in the modulating voltages, there will be a harmonic voltage at the output, and its amplitude will change as a function of delay according to the law of the envelope of the correlation function. In addition to the regular component, output

Table 2.1.

x_1	x_2	Output voltage
+1	+1	+1
-1	-1	+1
+1	-1	-1
-1	+1	-1

voltage also contains a variable component, whose intensity decreases as the correlation coefficient increases between the studied voltages.

The system described above is advantageous in that test results are independent of the characteristics of nonlinear elements, work is possible in a wide frequency band, and the system is simple and dependable.

Balanced functional correlator systems which are used to study narrow-band random signals and pseudorandom signals in the radio frequency range should be created in the form of resonance devices (Fig. 2.7), whose operational principle is analogous to the one discussed above. In the superhigh frequency range the simplest correlator could be made, for example, in the form of a balanced mixer based on a double-T connection or similar arrangements.

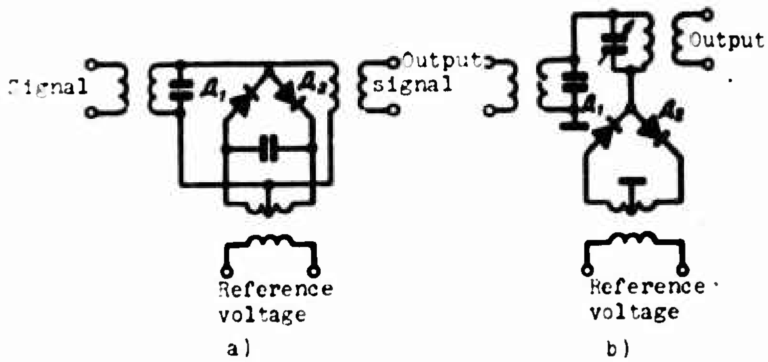

Fig. 2.7. Simplified functional correlator system for analyzing narrow-band random processes in the radio frequency range.

Electronic correlator systems are extremely diverse and deserve a special discussion.

2.4. CORRELATION DEVICES FOR TRACKING A SIGNAL DELAY

An example of a system which tracks a delay might be a target tracking radar station with correlation processing of the received signal.

Necessary elements in a tracking system are the correlation device (discriminator), which generates an error signal according to a controlled parameter, and a feedback circuit. Let us examine some examples of correlation devices which track a signal delay.

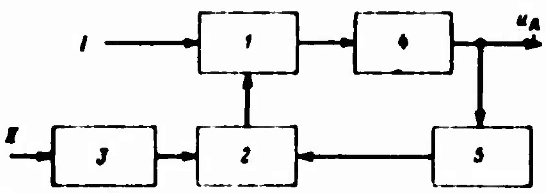

Fig. 2.8. Block diagram of tracking discriminator for noise signal delay: 1 - multiplier; 2 - controlled delay line; 3 - differentiation device; 4 - low frequency filter; 5 - feedback circuit.

Figure 2.8 [54, 57, 81] shows the block diagram of a discriminator operating under a noise signal.

The signal, containing information about the parameter which is being automatically tracked is transmitted to the discriminator along channel I along with noise

$$u_1(t) = A \cdot u_c(t - \tau_p) + u_m(t), \qquad (2.13)$$

where τ_p is delay time and A is the coefficient which describes signal attenuation.

Reference voltage $u_0(t) = u_c(t - \tau_0)$ is conducted to input II. This voltage passes through the differentiating block and through the line with controlled delay τ_0, which may differ from the actual delay τ_p by a small quantity $\varepsilon(t) = \tau_p(t) - \tau_0(t)$.

The dependence on t indicates that τ_p and τ_0 change in time during the working process of the system. We will assume that during the averaging time these changes are negligibly small.

Let us find the voltage at the discriminator output. Let us represent the signal in channel I in the Taylor sèries:

$$u_1(t) = A\left[u_c(t-\tau_0) + \varepsilon u'_c(t-\tau_0) + \right.$$
$$\left. + \frac{\varepsilon^2}{2!}u''_c(t-\tau_0) + \dots\right] + u_{ш}(t).$$

Prime signs indicate a derivative with respect to ε. If we assume that averaging time is great in comparison to correlation time and that the ergodic hypothesis is valid, then we get the following expression for voltage at the discriminator output:

$$u_д(t,\varepsilon) = \overline{A u_c(t-\tau_p)\frac{du_c(t-\tau_0)}{dt}} + \overline{u_{ш}(t)\frac{du_c(t-\tau_0)}{dt}}.$$

The second term determines extraneous noise, which is not analyzed. For the first term, if we put the expression for the input signal in the form of the Taylor series, we get

$$u_{д0}(t,\varepsilon) = B\{\overline{u_c u'_c} + \varepsilon\,\overline{(u'_c)^2} + \varepsilon^2\overline{u'_c u''_c} + \dots\}. \qquad (2.14)$$

where $u_c = u_c(t-\tau_0)$, $u'_c = \frac{du_c}{dt}$, $B = \text{const}$

The second term, whose sign and quantity are determined by the deviation in the actual delay signal from that established in the line, has the basic meaning. The error signal, which is dependent on difference $\tau_p - \tau_0$, acts upon controlled delay line 2 through feedback circuit 5. Delay tracking accuracy depends on a number of factors, including noise intensity, the amplification coefficient in the feedback circuit (block 5), and the discriminator characteristic, i.e., the form of function (2.14). The discriminator characteristic can have different forms. Figure 2.9 shows the discrimination characteristics for noise signals of two types: narrow-band and broad-band.

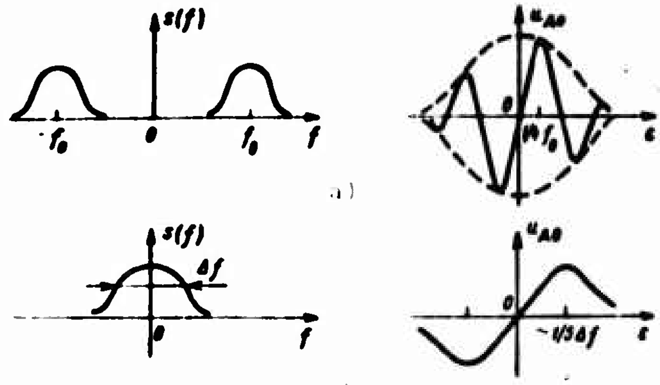

Fig. 2.9. Discriminator characteristics for noise signal delay: a) narrow-band noise; b) broad-band noise.

For narrow-band noise the working section of the discriminator characteristic corresponds to a noise signal lying in the range $-\frac{1}{4f_0} < \varepsilon < \frac{1}{4f_0}$. The working section of the characteristic has a positive slope. For narrow-band noise there may be several sections with a positive slope in the characteristic. This means that the system can work in a control mode at several values of τ_0 (in measuring systems this leads to error in determining the distance to target). For wide-band noise there is only one working section in the characteristic. In the case of small error signals the discriminator characteristic can be considered linear.

The discriminator can be created not only for a noise signal, but for signals formed on the basis of a pseudorandom sequence [57] (for example, an m-sequence). Figure 2.10 shows a block diagram of such a discriminator. The signal in the form of (2.13) is conducted through channel I to the discriminator. Voltages corresponding to two delays are conducted to the multipliers from storage system 4: $u_c(t - \tau_0 - t_0)$ - along channel IIa, $u_c(t - \tau_0 + t_0)$ - along channel IIb, where τ_0 is the expected delay time (controlled parameter), expressed as the number of symbols in the sequence. Voltages from the output of the multipliers are calculated and are averaged by means of filter 3 (averaging time is much greater than correlation time). Voltage at the output of filter 3 is determined by the relationship

$$u_\Delta(t,\varepsilon) = B[\overline{u_c(t-\tau_p)u_c(t-\tau_0-t_0)} - \overline{u_c(t-\tau_p)u_c(t-\tau_0+t_0)}] + R_m. \quad (2.15)$$

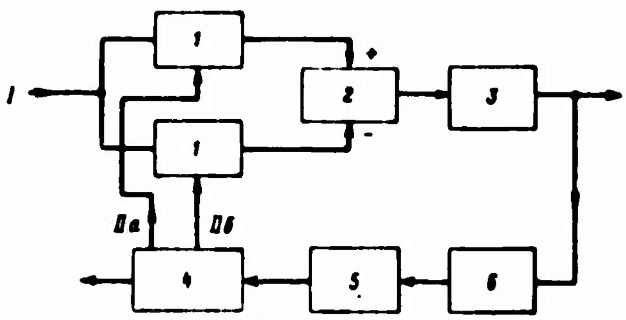

Fig. 2.10. Block diagram of tracking discriminator for delay in pseudorandom sequence: 1 - multipliers; 2 - adding system; 3 - low frequency filter; 4 - shaping and delay of pseudorandom sequence; 5 - delay control block; 6 - feedback circuit.

The last term describes the effect of noise for which no analysis is made. Let us examine the first term

$$u_{\Delta 0}(t,\varepsilon) = B[R(\varepsilon - t_0) - R(\varepsilon + t_0)], \quad (2.16)$$

where $R(\tau)$ is the cross-correlation function of the signal and the reference voltage.

Relationship (2.16) determines the discriminator characteristic which is shown in Fig. 2.11. The working segment of the characteristic corresponds to $-t_0 < \varepsilon < t_0$; when $|\varepsilon| > 2t_0$ output voltage is equal to zero. Whereas the studied signals represent a periodically repeated m-sequence, the discriminator characteristic is a periodic function of error signal ε. The characteristic depicted in Fig. 2.11 corresponds to an ideal pseudorandom sequence and thus contains breaks. The actual characteristic would not contain breaks because the pass bands of the linear blocks in the system are finite.

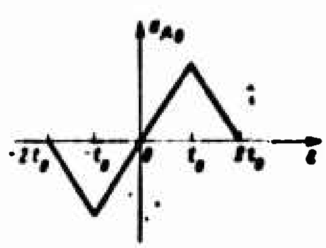

Fig. 2.11. Discriminator characteristic of pseudorandom sequence delay.

The voltage of the error signal from the output of filter 3 acts through feedback circuit 6 on delay system 4, changing τ_0 in accordance with the change in τ_p.

2.5. OPTICAL CORRELATOR

An operation corresponding to a correlation integrator can be performed not only by electronic systems, but by means of optical devices [53, 69, 75] which use both coherent and noncoherent radiation. Let us dwell on coherent optical devices. The use of optical devices in creating correlators is based on the use of these devices to perform multiplication and integration operations. In this case we use the property of the change in the amplitude and phase of the light as it passes through an optically heterogeneous medium and the focusing effect of the lenses.

An optical system can be easily analyzed using the method from the theory of the field and complex shape presented by an electromagnetic oscillation. The optical system will be completely (or three-dimensionally) coherent if the wave fronts represent a surface of both constant phase and constant amplitude. For a parallel, homogeneous light beam (a laser beam, for example,) we have $E_0 = \exp(i\omega t)$, where ω plays the role of the carrier. A coherent light wave of uniform amplitude is represented in the form of

$$E = A(x, y)\, e^{i\varphi(x, y)}.$$

The phase factor $\exp(i\omega t)$ is dropped, since the system is coherent, i.e., the phase relationships for light waves at different points in space are not time dependent.

The studied signal can be taken on a transparency. The transmittivity of the transparency describes the amplitude, for example, while its thickness describes the phase. The filters may also be made in the form of transparencies.

The transmittivity and thickness of the transparency are described by functions $t^2(x, y)$ and $\frac{\varphi(x, y)}{2\pi(n-1)}$, respectively, where

n is the refractive index of the material. After passing through the transparency, the light wave undergoes an amplitude change in accordance with the type of function $t^2(x, y)$ and additional phase shifts according to function $\alpha(x, y)$. A transparency with the indicated parameters can be described by the function

$$S(x, y) = t(x, y)e^{i\alpha(x, y)}. \qquad (2.17)$$

As the light wave travels through the modulating medium (served by the transparency) the following complex multiplication operation is performed

$$E_2 = S_1E_1 = A(x, y)t(x, y)e^{i[\varphi(x, y) + \alpha(x, y)]}. \qquad (2.18)$$

Indicators in the optical range (photocells and others) are sensitive to light energy, and thus their indications are proportional to the square of light wave amplitude. In recording light waves by means of complex functions, the expression for wave energy will have the form of

$$W = kEE^*,$$

where k is a constant coefficient; the asterisk denotes a complex-conjugate value.

Let us examine the system of an optical device by which the Fourier transform (Fig. 2.12) can be made. Lens $Л_1$ creates a parallel, monochromatic beam of rays. The spatial distribution of the field on plane P_1 will be designated as $E_1(x_1, y_1)$. Let us determine the distribution of the field on plane P_2 in systems of coordinates x_2 and y_2.

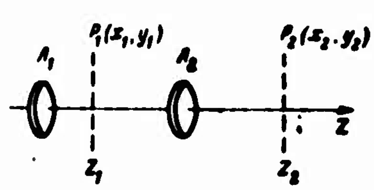

Fig. 2.12. Optical system which performs Fourier transform: $Л_1$, $Л_2$ - ideal lenses.

Analysis of the processes is based on the use of the Huygens principle. A complex representation of the field at point x_2, y_2 on plane P_2 is obtained as the sum of the components of all points on plane P_1. In a direction close to the axis of the optical device the voltage of the electromagnetic field created by the element of plane P_1 at the point with coordinates x_2 and y_2 is determined by the relationship

$$dE_2 \approx \frac{-AE_1(x_1,y_1)}{i\lambda a} \quad \frac{a(x_1,y_1,x_2,y_2)}{\lambda}\Big] dx_1 dy_1,$$

where A = const; ϕ_1 is the initial phase, a - optical distance between points with coordinates x_1, y_1 and x_2, y_2; λ - wavelength.

The differences in phases constituting the field in plane P_2 are interesting, and thus we can drop the factors which describe additional phase shifts which are common to all points. Attenuation in the field during wave propagation can also be ignored. The voltage of the field at point x_2, y_2 can be determined by the relationship

$$E_2(x_2,y_2) = \iint_{P_1} E_1(x_1,y_1) e^{-i\frac{2\pi}{\lambda} a(x_1,y_1,x_2,y_2)} dx_1 dy_1. \qquad (2.19)$$

Now let us calculate the quantity $a(x_1, y_1, x_2, y_2)$.

We will draw a straight line from point x_2, y_2 through the center of the lens to the intersection with plane P_1 at point x_0. Plane P_n is perpendicular to the straight line and forms the angle θ, whose normal is toward the axis of the system (Fig. 2.13). The plane wave whose phase front lies in plane P_n will be focused at the point with coordinate $x_2 = f \operatorname{tg} \theta$. The optical length of the path between the points of plane P_n and the point x_2 is equal to the constant value C, where

$$C = r_1 + r_2 = \sqrt{g^2 - x_0^2 \cos^2\theta} + \sqrt{f^2 + x_2^2}.$$

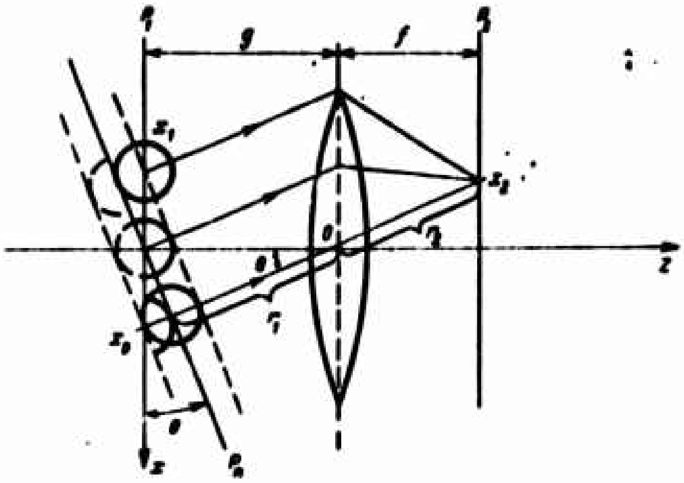

Fig. 2.13. Path of rays in lens.

Let us represent the terms in the first part in the form of a series and limit ourselves to the first two terms. Let us also assume that $x_0/g = x_2/f$, where f and g are the front and rear focal distances of the lens.

For small angles θ

$$C = g + f + \left(1 - \frac{g}{f}\right)\frac{x_2^2}{2f}.$$

According to the Huygens-Fresnel principle all points on plane P_1 located to the right of the studied plane P_n, i.e., along the propagation path of the light wave, should participate in the formation of the image at point x_2. Points to the left of P_n in this case should not be considered, since oscillations from secondary (virtual) emitters in the plane are damped by the preceding echelon of waves eminating from the wave front to the left of P_n. Physically this denotes the absence of waves to the rear.

The distance from the points on plane P_1 to point x_2, y_2 is found by adding to C the term $-x_1 \sin \theta$. For small angles θ $\sin \theta = x_2/f$ is valid.

As a result we get

$$a(x_1, x_2) = g + f + \left(1 - \frac{g}{f}\right)\frac{x_2^2}{2f} - \frac{x_1 x_2}{f}.$$

In the case of a three-dimensional space, the expression for distance a will be

$$a(x_1, y_1, x_2, y_2) = \text{const} + \left(1 - \frac{g}{f}\right)\left(\frac{x_2^2 + y_2^2}{2f}\right) - \\ - \frac{x_1}{f}x_2 - \frac{y_1}{f}y_2. \qquad (2.20)$$

The voltage of the field on plane P_2 is determined with an accuracy up to a constant factor by the relationship

$$E_2 = \left[\iint_{P_1} E_1(x_1, y_1)\, e^{i\frac{2\pi}{\lambda f}(x_1 x_2 + y_1 y_2)}\, dx_1 dy_1\right] e^{i\beta(x_2, y_2)}, \qquad (2.21)$$

where

$$\beta(x_2, y_2) = \frac{\pi}{\lambda f}\left(1 - \frac{g}{f}\right)(x_2^2 + y_2^2).$$

When $g = f$ the factor behind the brackets in formula (2.21) is equal to one. In this case relationship (2.21) represents the inverse Fourier transform. In the theory of the Fourier transform, in shifting from a frequency range to a time range, the kernel of the transformation has the form of $e^{i\omega t}$, i.e., its sign corresponds to the kernel in formula (2.21). Conjugate kernel $e^{-i\omega t}$ is essential in shifting from a time range to a frequency range. In the case of an optical transformation the sign in the kernel can be changed if the direction of the coordinate axes of the transformed function system is changed. Figure 2.14 shows the simplest optical device which can make forward and reverse Fourier transforms. The parallel beam of light rays from a coherent source falls to the left onto plane P_1. Quantity $E_1(x_1, y_1)$ describes the distribution of the field on plane P_1. The ideal lens, in which abberations are absent (abberations impair output resolutions, since they decrease integration effectiveness), performs an operation corresponding to a transform in the form of (2.21) when $\beta = 0$. If in (2.21) we designate

$$\omega_x = -\frac{2\pi x_1}{\lambda f} \text{ and } \omega_y = -\frac{2\pi y_1}{\lambda f},$$

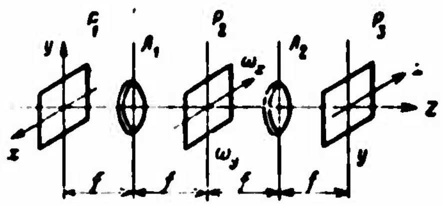

Fig. 2.14. Optical system which performs forward and reverse Fourier transforms.

then we get a formula which coincides with the forward Fourier transform (when the direction of the coordinate axes is changed), i.e., the kernel of the transform made by lens A_1 is equal to

$$\exp[-i(\omega_x x_1 + \omega_y y_1)].$$

Ideal lens A_2 performs an operation corresponding to the reverse Fourier transform. Here the direction of the coordinate axes in plane P_3 is the same as in plane P_2. The studied optical devices make it possible to perform transformations for both the x-axis and the y-axis. By nature the signals are functions of only one variable - time. In this case the second coordinate can be used for simultaneous processing of a great number of signals [53, 69]. The one-dimensional Fourier transform can be accomplished by means of a system similar to that shown in Fig. 2.14 by introducing cylindrical lenses which focus only in one plane. Cylindrical lenses must be introduced between the plane P_1 and the first lens and between plane P_2 and the second lens. Through the combination of cylindrical and spherical lenses a single [one-time] Fourier transform is achieved along the x-axis, and a double [repeated] transform along the y-axis. Plane P_2 in this case has coordinates ω_x, y.

The device which achieves the Fourier transform is the basis of the optical correlator shown in Fig. 2.15. Let us see how the cross-correlation function of two narrow-band random processes f(x) and g(x) are found. Spatial coordinate x can reflect time in a certain scale. A transparency with variable transmittivity, corre-

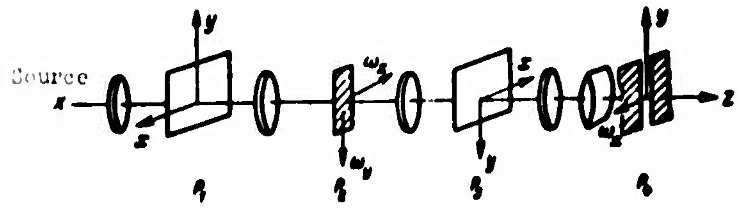

Fig. 2.15. Simplified optical correlator system.

sponding to function $f_0 + f(x)$ is placed in plane P_1, where f_0 is the reference level, relative to which the transmittivity of the transparency changes in accordance with function $f(x)$. The introduction of a reference [base] level is necessary, because the transparency cannot have a negative transmittivity value and is capable of reflecting only unipolar signals. Radio signals are bipolar. When a reference level is present the reading is begun at f_0 and the polarity of the signal is reflected. In plane P_2 we get the Fourier transform:

$$S = \iint_{P_1} [f_0 + f(x_1)] e^{i(\omega_x x_1 + \omega_y y_1)} dx_1 dy_1.$$

The presence of the reference level in subsequent multiplication operations may lead to additional parasitic background. To remove the background in plane P_2 a filter - a nontransparent plate, which does not transmit the spectral components of the signal f_0 and suppresses the carrier frequency component on the optical axis of the system - is introduced. In plane P_3 function $f(x)$ is formed, which is superimposed on the transparency which determines the second studied function $g_0 + g(x - x_0)$, where $x_0(\tau)$ is the space shift in the second transparency, which corresponds to a time shift by quantity τ in the second studied process. A product in the form of $g_0 f(x) + f(x)g(x - x_0)$ is formed. In plane P_4 we get the Fourier transform:

$$S_1 = \int_{l_x} g_0 f(x) e^{-i\omega_x x} dx + \int_{l_x} f(x) g(x - x_0) e^{-i\omega_x x} dx. \qquad (2.22)$$

A cylindrical lens integrates only for one axis. The slit in plane P_4 filters out the spectral components determined by the first term in (2.22) and separates the components of the second term when $\omega_x = 0$, which corresponds to the cross-correlation function of processes f(x) and $g(x - x_0)$, i.e.,

$$R(\tau) = \int_l f(x) g[x - x_0(\tau)] dx. \qquad (2.23)$$

If $g(x - x_0) = f(x - x_0)$, then we get the autocorrelation function. The integral of (2.23) corresponds to expression (1.9) for the correlation function, but is obtained by optical devices. An output lens (not shown in Fig. 2.15) is placed behind plane P_4, and projects the signal from the output slit to the indicator.

Optical devices make it rather simple to create multichannel correlators. In this case the transparency, which is placed in plane P_3, contains a large number of recorded signals, which are distributed along the vertical (along the y-axis). In detection systems these recordings may correspond, for example, to different Doppler frequency shifts or different signal delays. A filtered-out light distribution, corresponding to function f(x), is projected onto all of these signal recordings, i.e., the product of studied signals is formed in each channel. The next lenses form a separate integral transform for each channel. In plane P_4 the images are focused such that the channel division is preserved.

In optical signal processing the number of channels can be very great, which indicates the promising future of optical correlators.

THIRD CHAPTER

CORRELATED SYSTEMS

Our definition of a correlated system is a system based on the correlation method of processing signals. Almost all radio-electronic systems process useful signals in the presence of interference. The value of this interference in most cases cannot be predicted, and thus it is virtually impossible to completely eliminate harmful effects. In radioelectronic systems an attempt is being made to achieve a method of processing useful signals in which the effect of interference will be minimal. One of these signal processing methods is the correlation method.

Let us look at some examples of different types of correlated systems.

3.1. CORRELATED DETECTION SYSTEMS

The purpose of any radar or sonar system is to obtain information on the location of targets in the surrounding area and on their current coordinates. This information in an active locating method can be obtained by analyzing signals reflected from targets.

The main purpose of the locator is to determine the actual presence of a target in a certain region. In view of the random nature of inherent noise, the essence of our information, obtained

by processing an additive mixture of reflected signals and interference, is the probability value for the presence (or absence) of a target in surrounding space.

If interference is "white" gaussian noise [5, 8, 40], then this probability is a function of the cross-correlation value of the integral

$$R(\tau)=\frac{1}{T}\int_{0}^{T}u_c(t-\tau)u_{вх}(t)\,dt,$$

where $u_c(t - \tau)$ is a delayed copy of the probing signal; $u_{вх}(t)$ is the additive mixture of the received reflected signal and interference.

The receiver which calculates the cross-correlation integral retains all information contained in the received mixture of signal and noise. This type of optimal receiver can be achieved by means of correlators and coordinated filters.

The delay lines or shift-register devices (see § 1.3) may serve as the storage system for the probing signal, which is the basic element of the correlated system. If the delay value is changed, then different elements of the space can be scanned. The simultaneous scanning of several elements is possible if a set of correlators are used, which operate with reference signals of different delays, i.e., in a multichannel system.

Another method which enables optimal signal reception consists of using coordinated filters. The pulse characteristic of such a filter is coordinated with the probing signal according to the relationship

$$h(t)=au_c(t_0-t).$$

where a is a constant factor, t_0 - length of signal delay in filter; $u_c(t)$ - probing signal.

Voltage at the output of the coordinated filter

$$u_{\text{вых}}(t) = \int_{-\infty}^{\infty} u_{\text{вх}}(s)\, h(t - s)\, ds$$

is a time function, which repeats the cross-correlation function of the probing signal and the processed mixture of the reflected signal and interference.

Each of these optimal reception methods has its advantages and disadvantages. The main disadvantage of correlated systems is that they require successive scanning of range elements; a simultaneous sweep of the entire distance is only possible with multichannel signal processing. The basic advantages of correlated systems include the following:

1) It is possible to use signals which have a large spectrum width-storage time product. This is possible because storage time in a correlated system is determined by integrator parameters and is not related to signal delay.

2) The correlator is simple as compared to the coordinated filter. The more complex the signal which is used, the greater will be the advantage.

3) It is possible to use noise and pseudonoise signals whose structures can be varied as the system works.

The quality of detection is described by two probabilities: correct detection D and false alarm F. Usually the false alarm probability is fixed at a rather low level. In that case the correct detection probability will be a nondiminishing function of the parameter q.

In the case where a reflected signal is detected in the presence of "white" gaussian noise

$$q = \sqrt{\frac{2E}{S}},$$

where E is the energy of the received signal and S is the spectral density of gaussian noise.

The possibility of detection depends solely on the ratio of the energy of the useful signal to the spectral density of noise and does not depend on other signal parameters (for example, on spectrum width, steepness of pulse, etc.).

In the presence of a large number of targets in surrounding space and of intense reflections from local objects, the problem of distinguishing useful reflected signals from undesirable signals and several useful signals from one another is very important. The discrimination, parameter measurement accuracy of the reflected signal, and ambiguity of the measurements are described by the structure of the uncertainty function of the probing radio signal.

The uncertainty function can be determined by different methods: the inverse probability method, the probability function, or by the method of mean square error [8, 39, 40]. For example, if we take the absolute value of the average square value of difference ε^2 between two complex signals (reference signal and signal reflected from target), then we can write

$$\varepsilon^2 = \int_{-\infty}^{\infty} |u_c(t) - u_c^*(t-\tau)|^2 dt.$$

Here $u_c(t)$ is the reference signal

$$u_c(t) = u_M(t) e^{i2\pi f_0 t},$$

where $u_M(t)$ is the complex modulating function.

The signal $u_c^{fr}(t - \tau)$ reflected from the target differs from the stored signal by a delay of time τ and a Doppler spectrum shift by qua[illegible]:

$$u_c^{fr}(t-\tau) = u_M(t-\tau)\, e^{i2\pi(f_0 - f_g)(t-\tau)}.$$

In the expanded form we can write

$$\varepsilon^2 = \int_{-\infty}^{\infty} |u_M(t)|^2\, dt + \int_{-\infty}^{\infty} |u_M(t-\tau)|^2\, dt -$$

$$-2\,\mathrm{Re}\left[e^{i2\pi(f_0-f_g)\tau}\int_{-\infty}^{\infty} u_M(t)\, u^*_M(t-\tau)\, e^{i2\pi f_g t}\, dt\right],$$

where the symbol Re denotes the real part of the corresponding expression.

The first two terms represent signal energy; consequently, only the last term may serve as a measure of the mean square "difference" between the reference signal and the received signal. If we ignore terms of a higher order of smallness in the last term of the equation, then we get an uncertainty function in the form of

$$R(\tau, f_g) = \int_{-\infty}^{\infty} u_M(t)\, u^*_M(t-\tau)\, e^{i2\pi f_g t}\, dt,$$

or, after the Fourier transform is used,

$$R(\tau, f_g) = \int_{-\infty}^{\infty} G^*_M(f)\, G_M(f - f_g)\, e^{i2\pi f\tau}\, df,$$

where $G^*_M(f)$ is the modulating voltage spectrum.

The shape of the body of uncertainty, which is bounded by plane τ, f_g, and the surface corresponding to the modulus of the

uncertainty function, is determined by the structure of the signal and can have a different configuration. The connection between the shape of the body of uncertainty and the reference characteristics of the signals can be explained as follows. From a determination of the uncertainty function it follows that if two signals of similar types must be discriminated, then their shift with respect to τ and f_g should be such that the uncertainty function reverts to zero (or does not exceed a certain given value). The presence of large bursts in the body of uncertainty in addition to the reference indicates that signals reflected from local objects or from targets with a highly effective reflection area may mask the useful echo signal and, consequently, hinder discrimination.

The uncertainty function characterizes the accuracy of measuring range and function which a given signal can provide. The greater the concentration of the body of uncertainty at the origin of the coordinates, the more accurate the measurement of delay time and the Doppler shift.

It can be shown [8, 39, 40], that

$$\frac{\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}|R(\tau, f_g)|^2 d\tau df_g}{|R(0, 0)|^2} = 1.$$

Consequently the normalized volume of the uncertainty body thus determined is a constant quantity which is independent of both the length of the signal and of the law of modulation of its amplitude or phase. Attempts to compress the body of uncertainty by selecting an appropriate law of modulation or signal shape can lead either to a distribution of the entire volume in a thin layer on a large surface or the appearance of a series of additional peaks on plane τ, f_g. For noise and noise-like signals the main spike is concentrated in the region $f_g = \tau = 0$, and its extension is approximately equal to $1/\Delta f$ along axis τ and $1/T$ along axis f_g.

To assure high discrimination and measuring accuracy it is essential that the body of uncertainty of the signal have a narrow peak at the origin of the coordinates, i.e., the signal should have a sufficiently effective length T and spectrum width Δf. Noise signals of great duration and spectrum width and regular signals based on pseudorandom sequences have a body of uncertainty with this type of configuration. The magnitude of product ΔfT is of great significance for the signal characteristic, and is called the signal base. The base of pulse signals which do not have additional modulation within the pulse is a quantity close to one. These signals are called simple, and they do not assure high combined discrimination with respect to delay and frequency.

Complex signals, which are often called wide-band signals, are characterized by a base which considerably exceeds one. These signals have been widely used in detection systems and in measuring the coordinates of targets [33, 60, 62, 79].

1. Radar Sets with a Noise Signal

Because of their wide band, constant emission, and random nature, noise signals have an uncertainty function which approaches the ideal. This assures satisfactory noise locator characteristics. Furthermore, noise locators can work under a low signal/interference ratio at the receiver input.

In order to detect a signal reflected from a target against the background of interference or to determine the distance to a target or the velocity of the target, noise radar sets employ the correlation processing method or its variations - the "anticorrelation method" and the method of beats.

Figure 3.1 shows a block diagram of a noise radar set used for the correlated processing method. Voltage from the source of wide-band noise 1 (a pseudorandom pulse train generator can also be used) enters linear filter 2, which forms a random process

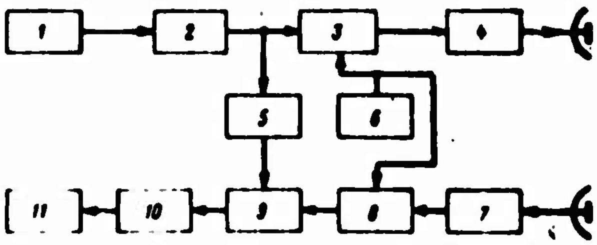

Fig. 3.1. Block diagram of correlated radar set with noise signal: 1 - wide-band noise source; 2 - linear filter; 3, 8 - transformers; 4 - power amplifier; 5 - delay block; 6 - heterodyne; 7 - low-noise amplifier of receiver; 9 - multiplier; 10 - narrow-band filter; 11 - gate.

with an autocorrelation function corresponding to a given discrimination. The formed signal enters transformer 3 and is transformed by means of the high-frequency harmonic voltage of heterodyne 6 into the range of working frequencies. After amplification and power amplifier 4 the signal enters the transmitting antenna and is emitted. The signal which is selected from the target passes through low-noise amplifier 7 and enters transformer 8. In order to achieve cross-correlation processing the modulating noise signal from the output of block 2 is delayed in block 5 by time τ_0 and enters the input of multiplier 9. The signal which has been received and transformed enters the second input of this multiplier together with the inherent noise of the receiving channel. The product of the signals is averaged by narrow-band filter 10 and is compared in block 11 to the threshold. If a stationary target is located at a range corresponding to a delay τ which is close to τ_0, then the voltage at the output of block 10 will have a value which is close to the maximal. When a corresponding threshold is selected, this will trigger block 11. When τ has a value which differs from τ_0 by more than the correlation time at the output of multiplier 9 there will be no voltage overshoot, even if there is a reflected signal. In the case of a single-channel system the correlated locator must scan range elements in succession. In this case total scanning time may be very great. The multichannel set is required to reduce this time. Here the number of multipliers and successive filtration blocks should be the same as the number of range elements scanned simultaneously. Thus, the arrangement of the set becomes complex.

In order to select the velocity of the target block 10 must contain a set of narrow-band filters whose central frequencies are shifted to the Doppler frequency discrimination interval. The number of filters is determined by the discrimination interval and by the entire range of central frequency shifts in the reflected signal due to the Doppler effect.

The use of a difference frequency correlator in a noise radar set makes it possible to improve the actual response and to select targets which lie at equal distances from the set and are the same in size, but have radio velocities of opposite directions. To achieve this regime reference voltages of different frequencies must be conducted to blocks 3 and 8 and the adjustment of the filters in block 10 must be changed.

The work of the noise locator in a constant emission mode or in detecting nearby targets is rather complex. In the case of the constant emission mode the emitted signal component enters the receiver. The intensity of this component, even with the most careful separation between transmitter and receiver channels, considerably exceeds the signal reflected from the target. In work involving nearby targets, penetrating emissions may correlate with the reference voltage in the receiver.

Some modifications of the correlated locator make it possible to decrease the effect of the emitted signal which penetrates the receiver; such modifications include the "anticorrelation" receiver and the "beats" method [77].

In the "anticorrelation" noise radar set the signal is processed as follows. A mixture of the received signal, noise, and the reference oscillation (part of the emitted signal serves in this role) enter the input of the subtracting device. The obtained difference is squared and integrated. The following voltage is formed at a output of the integrator:

$$u(\tau_0, T) = A\int_0^T [u_c(t) - ku_c(t-\tau_0) - u_ш(t)]^2 dt,$$

where A is a constant; k - ratio of the power of the received signal to reference voltage power; $u_c(t - \tau_0)$, $u_ш(f)$ - received signal and noise of receiver, respectively.

If noise has a low intensity and the correlation time of the probing signal is much smaller than integration time, then we get approximately

$$u(\tau_0) \approx A[(1+k^2)R_c(0) - 2kR_c(\tau_0)].$$

Output voltage depends not only on delay τ_0 but also on signal attenuation. If we assume that k = 1, we get

$$u(\tau_0) \approx 2A[R_c(0) - R_c(\tau_0)].$$

With an accuracy up to a constant factor this expression represents fulfillment of the autocorrelation function of the signal up to its maximal value. The appropriate power spectrum and bandwidth for the noise signal can be selected by the unique determination of distance directly from the value of the output voltage, which makes it possible to do without the controlled delay line. The dependence of output voltage on reflected signal intensity can be eliminated if the probing signal is shaped by modulating frequency according to the random law.

A variation of the correlation processing method is the beats method. In order to determine the range and velocity of the target beats are formed in the receiver between the received and emitted signals, which are compared with "local" beats, formed by a special system. If the signal cos $[2\pi f_0 t + \phi(t)]$ is emitted, then the received signals will have the form of

$$\cos[2\pi f_0(t-\tau_0) - 2\pi f_g t + \varphi(t-\tau_0)],$$

where f_g is the Doppler frequency change.

At the mixer output, to which the emitted and received signals are conducted, we obtain in the difference frequency range the so-called "received" beats

$$\cos[\pi f_g t + 2\pi f_0 \tau_0 + \varphi(t) - \varphi(t - \tau_0)].$$

A special system forms "local" beats, which are described by the function

$$\cos[\varphi(t) - \varphi(t - \tau)].$$

where τ is the delay time of the reference voltage relative to the emitted signal in the "local" beat shaping systems.

In mixing "local" and "received" beats we get

$$\cos[2\pi f_g t + 2\pi f_0 \tau_0 + \varphi(t - \tau) - \varphi(t - \tau_0)].$$

From this expression it follows that in the general case output voltage is a mixture of a harmonic signal of Doppler frequency and random noise. When $\tau = \tau_0$ the output signal becomes purely harmonic, which makes it possible to determine delay τ_0 and Doppler frequency f_g.

2. Radar Station with Pseudonoise Signal

The use of a pseudonoise signal in correlated locators is interesting primarily because of the fact that it is easier to generate and delay this signal than a noise signal [60, 79]. Figure 3.2 shows a simplified diagram of a continuous-wave short-range radar which uses an m-sequence generator to shape the probing signal. The set is based on the "beats" method, which was discussed in the preceding section.

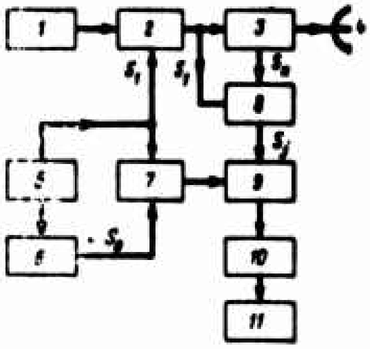

Fig. 3.2. Block diagram of radar set with pseudorandom signal: 1 - harmonic SHF generator; 2 - SHF modulator; 3 - circulator; 4 - antenna; 5 - pseudorandom signal generator; 6 - delay block; 7 - device for shaping "local" beats; 8 - mixer of receiver; 9 - correlator; 10 - narrow-band low-frequency filter; 11 - gate.

The pseudorandom video signal from generator 5 enters the superhigh frequency (SHF) modulator (block 2). The harmonic oscillation from the SHF generator (block 1) is supplied to the second input of the modulator. At the output of the modulator 2 a phase-manipulated signal is shaped. The oscillation phase jumps by π according to the law of a pseudorandom train. Through circulator 3, which divides the transmitter and receiver channels, the shaped signal is conducted to antenna 4 and is emitted.

The signal which is reflected from the target passes from the antenna through circulator 3 and enters the balance mixer of receiver 8. Part of the emitted signal is conducted to the second input of the balance mixer. Let us designate S_1 as the pseudorandom sequence which determines the modulation of the emitted signal. The received signal is modulated by pseudorandom sequence S_n, which differs from modulating sequence S_1 by a shift in the propagation time of the radiowave to the target and back.

At the output of the balance mixer a pseudorandom signal will be obtained which varies according to the law of the pseudorandom sequence $S_j = S_1 \oplus S_n$. The sum of the two m-sequences of a given polynomial will also be an m-sequence of the same polynomial, but with a time shift different from S_n (see § 1.3). The obtained beats are compared with "local beats," formed by adding sequence S_1 with the sequence corresponding to the expected propagation time of radio waves S_0. Formed and "local beats" are conducted

to correlator 9. When the expected and actual delays are equal, voltage at the correlator output is maximal in magnitude and changes in time according to the harmonic law with a frequency equal to the Doppler change in the frequency of the emitted signal. To reduce scanning time parallel analysis of the signal with respect to time (distance) as well as frequency (velocity) is necessary, i.e., arranging the set according to the multichannel system.

3. Generalized Block Diagram of Correlated Detection System

Different correlated detection systems can be represented approximately by the block diagram shown in Fig. 3.3. The filtration and transformation devices in actual sets can be divided into two groups: 1) devices which have the same effect on both the emitted signal and on stored reference voltage; 2) devices which affect only the probing signal or only the stored reference voltage. Devices of the first type in an equivalent system belong to general filter Φ_0. This includes primarily filters in the signal generator block. Devices of the second type are represented in the equivalent system in the correlator channels Φ_1, Φ_2. Filter Φ_1 includes the devices contained in the transmitter and receiver (before the correlator) as well as signal transformations occurring in radio

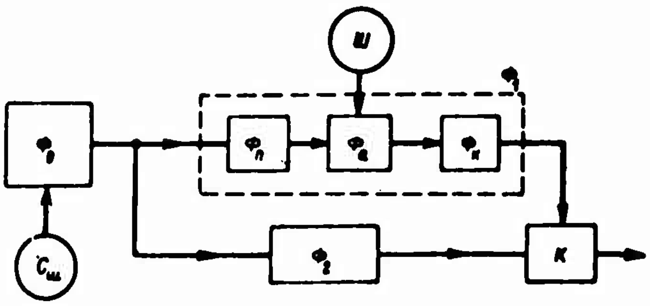

Fig. 3.3. Generalized scheme of correlated detection system: $C_ш$ - source of noise or noise-like signal; Φ_0 - filter in general channel; Φ_1 - filter in correlator channel; Φ_2 - filter in reference voltage channel; $\Phi_п$ - filter in transmitter; $\Phi_н$ - filter in receiver; Φ_a - filter which considers signal transformation during propagation; К - correlator; Ш - source of extraneous interference and internal noise of receiver.

wave propagation to and from the target and during reflection. Filter Φ_2 includes devices contained in the reference signal channel.

3.2. PASSIVE CORRELATED MEASURING SYSTEMS

In a number of cases the correlated system does not have emitted and reference signals. The action of the system is based on analyzing and measuring the characteristics of a signal created by an outside source. In this case the outside signal, which enters the correlator together with interference along one of the system channels, serves as reference voltage. These conditions occur, for example, in radar [30] in measuring fluctuations in SHF instruments [26], in passive radar, etc. Systems of this type will be called passive correlated systems, which emphasizes the fact that these systems do not emit.

The most general example of a passive correlated system is the correlated radiometer whose block diagram is shown in Fig. 3.4. The radiometer receives emission along two independent channels. The internal noise of the receiving channels is not correlated. The voltages of signals received by each channel are created by one source and are strongly correlated. Under these conditions multiplication of internal noise creates only a fluctuating voltage component at the output, while multiplication of input signals create along with the fluctuating component a regular voltage component, which is proportional to the value of the cross-correlation function of the input signals. Signals from a common source can enter the channels of the radiometer by being propagated along practically the same path or along two completely different paths. In the second case one of the radiometer channels should contain the delay line. The value of the output voltage, which is proportional to the maximal value of the cross-correlation function, can be established by changing the delay value. Under these conditions the coordinates of the radio emission source can be

determined if we know the magnitude of delay and the position of the radiometer antennas in space.

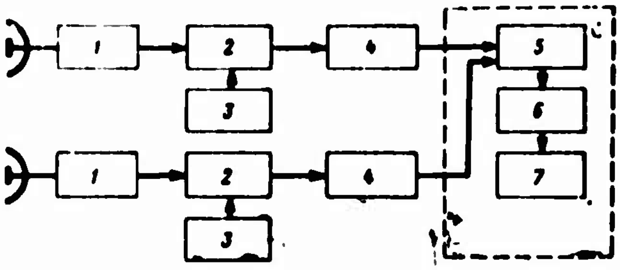

Fig. 3.4. Block diagram of correlated radiometer: 1 - high-frequency amplifier; 2 - mixer; 3 - heterodyne; 4 - intermediate frequency mixer; 5 - multiplier; 6 - filter; 7 - register.

To increase the response of the radiometer a difference frequency correlator should be used. In this case the spectrum of the input signal in one of the channels receives an additional frequency shift. Useful output voltage will be separated on a frequency equal to the frequency shift; voltage amplitude will be proportional to the envelope value of the correlation function. The averaging filter at the correlator output should be a band-pass filter with a central frequency equal to the frequency shift.

To increase the response of the radiometer and decrease false triggering we must have a good separation between channels. The fact is that the penetration of internal noise from one channel into the other will be registered at the radiometer output as a signal.

In correlated radiometers, in place of the usual analog correlators, functional correlators, for example, a polarity coincidence correlator (see § 2.1), can be used. The polarity coincidence correlator contains limiters in each channel. Oscillations from the radiometer receivers enter the inputs of these limiters. The output voltages of the limiters are conducted to the multiplier, whose response is averaged by means of a low-frequency filter. If limiting is sufficiently effective, then in place of an analog multiplication operation we can use logic multiplication, which is convenient when digital processing is used.

As a result of the limiting of input oscillations, which is done in the polarity coincidence correlator, the response statistics of the device in the absence of a useful input signal are determined solely by the statistics of interception by input oscillations of a certain level (most frequently zero). If the probability distribution density of processes at the outputs of both channels are symmetrical with respect to the Y-axis, then the level of false alarms will not depend on the probability distribution density of noise values [67]. In almost all cases the noise statistics correspond to these conditions, and thus the level of false alarms in a correlated radiometer containing a polarity coincidence correlator does not depend on the power of internal noise. The noise of the receiving part of the correlated radiometer has a nonstationary nature (due to the rotation of the antennas, fluctuation of amplifier coefficients, etc.), and thus the advantage noted above, which can be gained by using a polarity coincidence correlator, is useful.

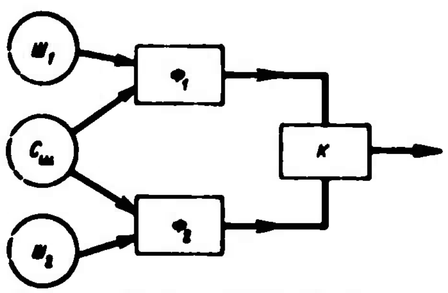

Fig. 3.5. Generalized correlated radiometer system.

Figure 3.5 shows a generalized block diagram of a correlated radiometer. The system contains only filters in correlator channels Φ_1 and Φ_2. The system does not contain general filter Φ_0, whose parameter selection would affect detection characteristics and measuring.

3.3. ANTIFADING COMMUNICATIONS

One factor hindering radio communications (particularly short-wave) is the multibeam propagation of radio waves. Here the transmitter signal arrives at the reception point along different propagation paths. Interference from signals with different beams

on the reception side leads to fading. Attempts to reduce the effect of signal interference have led to the development of information transmission methods in which complex signals and correlated processing methods are used.

Time discrimination of complex signals in correlated reception can be used successfully to increase the effectiveness of multibeam communications channels. If we consider the fact that the width of the correlation function of the signal is inversely proportional to the width of the spectrum, then for a complex signal ($\Delta fT >> 1$) we get

$$\tau_{\kappa} \approx \frac{1}{\Delta f} \ll T,$$

i.e., the correlation function is considerably shorter than information transmission T. At the output of the optimal receiving system there is a time compression in the signal. In using complex signals it is possible to completely separate time-overlapping signals, if the difference in arrival time of the various information transmissions is greater than the width of the compressed signals. This is the basis of using complex signals in communications channels which have multibeam propagation. The discrimination of time-overlapping signals makes it theoretically possible to divide the sum multibeam signal arriving at a reception point into individual beam-signals, which travel along each of the propagation routes. The separation of beams is complete if the correlation time of the signal is less than the least difference in propagation times of signal-beams. In the case of a greater number of beams the division is considered sufficient if the fading of each separated beam becomes small, i.e., the mixing effect of the fading will not be greater than the effect of additive fluctuation noise.

In separating beams it is theoretically possible to add the energy of the beams (not vector addition, as in ordinary systems).

Beam separation enables measuring the main parameters of each of the propagation paths of the signal. The results of the measurements can be used to create a self-adapting signal reception system.

Dependable reception of wide-band signals is possible when the power at the reception point is considerably less than noise power. Let us look at the dependence of the signal/noise ratio at the output of an optimal system for processing complex signals on the input signal/noise ratio:

$$\frac{P_c^{вх}}{P_ш^{вх}}=\frac{P_c^{вх}T}{P_ш^{вх}T}=\frac{1}{2\Delta f}\frac{2E}{S},$$

where Δf, T represent the effective width of the spectrum and length of the complex signal; S - spectral density of interference power.

The quantity $2E/S$ is the peak ratio of signal power to noise power at the output of the optimal system. Hence,

$$\frac{P_c^{вх}}{P_ш^{вх}}=\frac{1}{2\Delta f T}\frac{P_c^{max}}{P_ш^{max}}.$$

Thus, if the base of the complex signal is sufficiently great, transmission and dependable signal reception can be achieved. The power of the signal will be considerably less than that of internal noise or acting interference. This fact is important if we want the communications systems to be electromagnetically compatible and concealed.

There are many methods of creating correlation communications systems designed to operate with wide-band signals. We might single out two main types. Systems of the first type are based on the use of a group of wide-band signals. Corresponding to each

signal or combination of signals is a certain communication. Systems of the second type are based on the use of one wide-band signal, which represents the carrier oscillation. In order to transmit information the noise-like carrier oscillation is modulated by the usual modulation method (AM, FM, PM, changing τ, etc.).

The signals used in systems of the first type should have the necessary correlation properties. A coding sequence assembly should be used in which, first, cross-correlation between any pair of sequences would be as small as possible and, second, the autocorrelation function would not have significant lateral spikes. Different methods of creating and shaping such code sequence groups have been suggested. Noise signals and regular signals which follow definite laws can be used in such sequences. Shift register sequences of maximal length are widely used because they are easy to generate, can be independently reproduced on the transmitting and receiving sides, and are not difficult to synchronize (see § 1.3).

In systems of the second type wide-band signals of different types are used as the carrier oscillation. It is necessary only that the additional peaks of the correlation function for these signals be small. Otherwise false information could result.

The correlation approach of wide-[illegible] signals in communications systems can be achieved if the follo[illegible]ditions are fulfilled: 1) possibility of independent reprodu[illegible] of code transmissions on receiving side; 2) possibility of synchronizing code generators on transmitting and receiving side. A deviation from the above-mentioned conditions results in distortion and a decline in noise stability.

Let us examine a simplified block diagram of a communications system of the first type (Fig. 3.6). On the transmitting side codes of the selected group of signals are generated. Depending

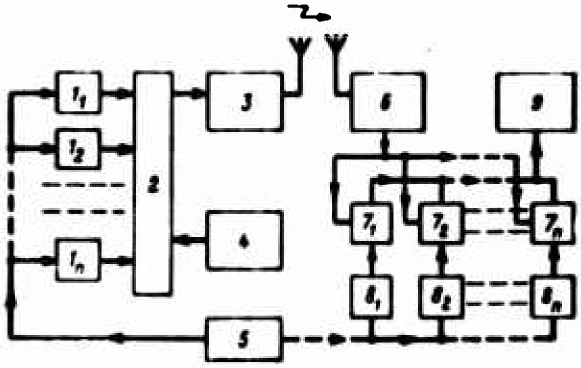

Fig. 3.6. Block diagram of communications systems with noise-like signals: $1_1 \ldots 1_n$ - generators for complex noise-like signals; 2 - manipulator; 3 - transmitters; 4 - communications source; 5 - synchronization; 6 - amplifier; $7_1 \ldots 7_n$ - correlators; $8_1 \ldots 8_n$ - generators for complex signals; 9 - resolution.

on the type of communications transmitted, the manipulator switches in an appropriate code generator or group of generators to the transmitter. On the receiving side of the set, after transformation in the high-frequency blocks, the signal enters the correlators. A reference signal is fed to each of the correlators from an appropriate code generator. At the correlator output, where correlation exists between the input and reference signals, the peak of the regular voltage component appears. In the absence of correlation at the correlator output there will only be noise voltage. The resolution device must decipher the transmission based on analysis of a received mixture of signal and noise.

The final value of the cross-correlation function between code sequences leads to specific noise, which is called the noise of nonorthogonality. The effect of the indicated noises is particularly great when the useful signal is received against a background of a large number of other signals belonging to the same code group. The second factor which results in decreased working efficiency of the communications system is a disturbance in the synchronous operation of code sequence generators on the transmitting and receiving sides.

Let us examine a simplified block diagram of a communications system of the second type, intended for transmitting analog information (Fig. 3.7). A noise or a pseudonoise carrier, obtained

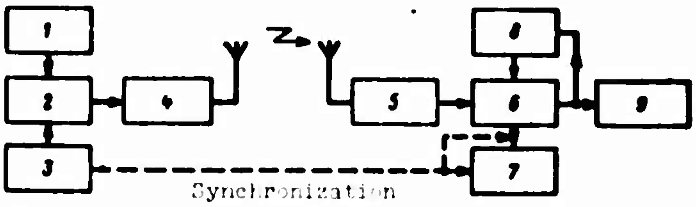

Fig. 3.7. Simplified block diagram of correlation communications system with pseudorandom carrier: 1 - source of communications; 2 - modulator; 3 - complex signal generator; 4 - transmitter; 5 - high-frequency receiver block; 6 - correlator; 7 - complex signal generator; 8 - system adjustment device; 9 - resolution.

by means of a complex signal generator, carries the communication. In order to transmit information the noise-like carrier oscillation is manipulated or modulated by means of standard modulating methods (AM, FM, PM, etc.). If a noise generator is used in the transmitter, then at the reception point an additional reference signal must be transmitted. This complicates the device and generally two identical pseudonoise signal generators are used in such communications systems in the transmitter and receiver. These generate identical, periodically repeated complex determined signals. In this case rigid synchronization of these generators is required and can be accomplished either by transmitting a synchronizing oscillation through a separate channel or by using special coding methods.

The system can also be designed so that the transmitter contains only one wide-band signal generator, for example, a noise generator, and the emitted signal then represents the sum of two signals - the signal at the generator output and the same signal delayed by time $\tau(t)$. The magnitude of delay depends on information concerning the communications being transmitted, i.e., delay modulation is performed. On the receiving side of the system there is autocorrelated reception of the signal, i.e., there is no wide-band reference signal in the receiver.

Figure 3.8 shows a generalized scheme for a correlated communications system. The system contains two synchronized signal sources -- in the receiver and transmitter, respectively. If we assume that ideal synchronization of signal sources is present, then the block diagram can be represented in the form shown in Fig. 3.3, where the filter transmission coefficient Φ_0 is equal to one. The source of the communication is not shown in Fig. 3.8. It is assumed that the generator of signal C_1 was selected according to the communication being transmitted, while on the receiving side only the generator whose signal is correlated with the transmitted signal is working.

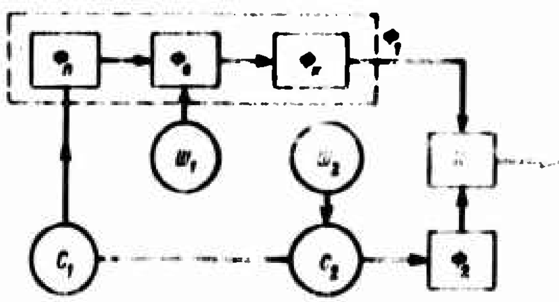

Fig. 3.8. Generalized scheme of correlated communications system. C_1, C_2 - complex signal generators in transmitter and receiver; Φ_1, Φ_2 -- filters in correlator channels; K - correlator, $\Phi_п$ - filter in transmitter; Φ_a - filter which considers distortion in signal during propagation; $\Phi_к$ - filter in receiver; $Ш_1$, $Ш_2$ -- source of noise in receiving channel and in synchronization channel.

3.4. CORRELATED MEASURING COMPLEXES

The cosmic radioengineering complexes of [2, 64] might serve as examples of correlated measuring complexes. The basic problems which can be resolved by means of cosmic complexes include the following: 1) measuring the coordinates and velocity of a cosmic target; 2) transmitting telemetric information to earth; 3) transmitting a control command. The solution to these problems

can be obtained by means of a modern radioengineering complex system. Such a combination makes it necessary to use special signals. Let us dwell briefly on certain peculiarities in solving just the first problem of those mentioned above. Measuring the movement parameters of a space vehicle is reduced to measuring range, rate of movement, and angular coordinates. The distance from the ground station to the space vehicle can be determined by measuring the propagation time of the radio signal. At great distances the power of the reflected signals is extremely small and can be considerably less than the power of the self-noise of the receiver. Thus, to assure steady reception, the signal should have high energy, i.e., the average permissible power should be of great duration. Furthermore, to eliminate uncertainty in measuring the propagation time it is desirable that the period of the probing signal be greater than the propagation time to maximal range. The pseudorandom sequences and phase-manipulated signals obtained from them, which were discussed above (see § 1.3), have the indicated properties.

The reflected signal is correlated with the copy of the emitted signal which is in the receiver (reference signal). Since the generated sequence can be very long, the main problem becomes one of synchronizing the reference signal with the received signal. To assure synchronization a step correlation method can be used, which consists of the following. A certain phase of the reference sequence is selected (i.e., the work of the shift register which forms the reference sequence begins when the cells are in a certain state) and is determined by the presence of correlation between the reflected and reference signals. In the absence of correlation the next value of the reference signal phase is tested, etc. The number of tests necessary to assure synchronization in the studied signals may be equal to the number of symbols in the sequence, i.e., may be extremely great. Storage time in analyzing each test can also be great. The step correlation method may prove unsuitable because of the great amount of time required to establish synchronization between the signals.

At the same time from the standpoint of the theory of information for assuring signal synchronization we need $\log_2 N$ tests. It is possible to approach this limit by using combined sequences (distance-measuring codes) to create an emitted signal (see § 1.3). The combined sequences are formed of several m-sequences by means of logic operations. The autocorrelation function of distance-measuring codes has several peaks of different sizes, which are used to determine the phase of each of the components in the combined sequence. The order of operations may, for example, be as follows. The received signal is compared with one of the component short sequences and a phase is established which corresponds to maximal correlation (if this occurs). Then, comparison is made with another component sequence, etc. The successive steps assure synchronization between the received and reference signals. The maximal value of the correlation function corresponds to complete synchronization. This comparison procedure which is done by means of combined codes, enables us to synchronize signals after fewer tests than would be required in step correlation.

One of the stages in this successive comparison of signals is that of determining the frequency of the reflected signal and measuring its Doppler shift. To facilitate the signal comparison procedure a rigid relationship is established between the carrier frequency of the emitted radio signal and the clock frequency of the pseudorandom code [64].

Determining the angular coordinates of an object in space is done by means of either one multibeam antenna or by the interferometric method. The work of the interferometer is based on measuring the difference in arrival time of signals from an object in space received by the main set and by two additional sets located a position distance from the main set. As an example Fig. 3.9 shows the block diagram of a correlated measuring system [64]. The master radar station emits a signal for which the

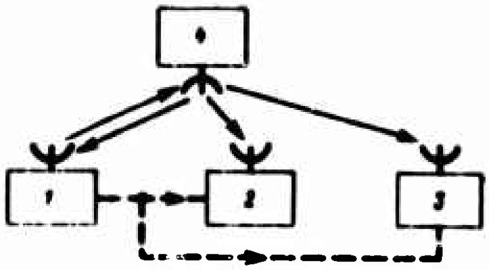

Fig. 3.9. Block diagram of correlated complex for measuring orbit parameters: 1 - master radar station; 2, 3 - slave radar stations; 4 - space vehicle; ---- synchronization line.

combined pseudorandom sequence serves as the modulating voltage. The space target re-emits the received signal from the ground station. The answer signal is captured by the receiving antennas of the master station and the slave stations. The tracking discriminator of the master station constantly tracks the delay in the received signal and calculates the distance to the target and the target's velocity. By means of the tracking discriminators of the slave stations the difference between delays in the arrival of the answer signal at the master station in each of the slave stations is calculated. On the basis of these measurements the angular coordinates of the target are determined.

FOURTH CHAPTER

NOISE CHARACTERISTICS OF FUNCTIONAL DIFFERENCE FREQUENCY CORRELATORS ON NONLINEAR ELEMENTS OF THE ν-th POWER

4.1. USE OF TRANSFORMATION METHOD TO ANALYZE NOISE CHARACTERISTICS OF FUNCTIONAL CORRELATOR

The work algorithm of a functional correlator is determined by relationships (2.9) and (2.10), in which under the integral sign we find, in place of the product of the studied random processes, another function of these processes. A functional dependence can be formed by means of nonlinear elements and devices. It is possible to represent a simplified block diagram of a functional correlator, consisting of a nonlinear element, an averaging filter, and an indicator.

The function which approximates the characteristic of the nonlinear device should be universal, i.e., when the individual parameters are changed it should correspond to the characteristics of the different nonlinear devices. In our opinion good results can be obtained by approximating the volt-ampere characteristics by power or exponential functions. Certain types of functional dependences, along with the simplest functional correlator systems, are given in § 2.3. In analyzing the noise characteristics of functional correlators the half-wave characteristic of the ν-th power should be considered:

$$z = \begin{cases} ay^{\nu} & \text{при } y \geq 0, \\ 0 & \text{при } y < 0. \end{cases} \qquad (4.1)$$

[при = when]

where ν is a positive real number, y - input voltage.

When $\nu = 1$ or $\nu = 2$, this represents a linear or a quadrat[illegible] correlated detector, respectively. If we form systems of seve[illegible] nonlinear elements, then we obtain nonlinear devices with full wave characteristics corresponding to the even

$$z = \begin{cases} ay^{\nu} & \text{при } y > 0, \\ 0 & \text{при } y = 0, \\ a(-y)^{\nu} & \text{при } y < 0, \end{cases}$$

and uneven

$$z = \begin{cases} ay^{\nu} & \text{при } y > 0, \\ 0 & \text{при } y = 0, \\ -a(-y)^{\nu} & \text{при } y < 0 \end{cases}$$

The transformation when $\nu = 0$ and subsequent multiplication of the studied processes correspond to the work algorithm of the polarity coincidence coordinator.

In this chapter we study the functional correlator, which is based on a nonlinear unit with a half-wave characteristic of the ν-th power. The regular component of the input voltage which describes the correlation connection between the studied random processes is found, along with noise intensity at the output of the filter which follows the nonlinear element. The noise characteristics of the functional correlator are compared with the analogous characteristics of an ideal correlator.

Processes in functional correlators can be easily analyzed by the transformation method [14, 24, 35], since the characteristics of nonlinear devices can be represented in the form of contour integrals. Let $z = g(y)$ be the characteristic of the nonlinear

transformation; $g(y) = 0$ when $y < 0$, and increases as y increases, but no faster than the exponential function, i.e.,

$$|g(y)| < Me^{u_1 y}. \tag{4.2}$$

where M, u_1 are constants.

In this case function $\gamma(y) = g(y)e^{-u'y}$, where $u' > u_1$, is absolutely integrable, i.e.,

$$\int_{-\infty}^{\infty} |g(y)\, e^{-u'y}|\, dy < +\infty. \tag{4.3}$$

and the Fourier transform exists

$$\gamma(y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ivy} \left[\int_0^{\infty} g(\xi)\, e^{-(u'+iv)\xi}\, d\xi \right] dv.$$

If we multiply out the left and right parts of the equality by $e^{u'y}$, then we get:

$$g(y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{(u'+iv)y}\, dv \left[\int_0^{\infty} g(\xi)\, e^{-(u'+iv)\xi}\, d\xi \right].$$

Let us introduce the complex variable $w = u' + iv$. The integral from the actual variable can be represented in the form of a contour integral on the plane of the complex variable along line $w = u' + iv$, i.e.,

$$g(y) = \frac{1}{2\pi i} \int_{u'-i\infty}^{u'+i\infty} f(w)\, e^{wy}\, dw;\ u' > u_1. \tag{4.4}$$

where

$$f(w) \doteq \int_0^{\infty} g(y)\, e^{-wy}\, dy. \tag{4.5}$$

The transition function of the nonlinear device f(w) in this case represents the unilateral Laplace transform of the characteristic. Output voltage g(y) is determined in the form of the reverse Laplace transform [(4.4)].

Cases are possible in which the characteristic of the nonlinear transformation does not revert to zero on the half-plane, but at positive and negative values of y increases no faster than the exponential function. In these cases the characteristic should be represented in the form of the sum of the two half-wave characteristics as in (4.1), which are valid for positive and negative values of the input signal.

In analyzing the noise characteristics of functional correlators let us consider the assumptions made in studying an ideal correlator (see § 2.2) to be valid. In addition we will assume that:

1. The signal received $u_c(t + \tau_з)$, reference voltage $u_г(t)$, and extraneous noise $u_ш(t)$ enter the input of the nonlinear device additively.

2. The averaging filter, which follows the nonlinear unit, is not coupled to it and therefore, does not have a reverse effect on the transformation pr

The input voltage which enters the nonlinear element represents a sum of the following form:

$$y(t) = u_г(t) + u_c(t + \tau_з) + u_ш(t). \tag{4.6}$$

The autocorrelation function of this voltage has the form of:

$$\begin{aligned} K_y(t, \tau) = {} & \overline{u_г(t) u_г(t+\tau)} + \overline{u_c(t+\tau_з) u_c(t+\tau+\tau_з)} + \\ & + \overline{u_г(t) u_c(t+\tau+\tau_з)} + \overline{u_c(t+\tau_з) u_г(t+\tau)} + \\ & + \overline{u_ш(t) u_ш(t+\tau)}. \end{aligned} \tag{4.7}$$

If we consider (2.4) and (2.5b), then we get the following expression for the autocorrelation function of the voltage at the input of the nonlinear element

$$K_y(t,\ \tau) = \sigma_r^2 \rho(t,\ \tau). \tag{4.8}$$

In expression (4.8) we have

$$\begin{aligned} \rho(t,\ \tau) = {} & r_r(\tau)\cos\omega_r\tau + \frac{\sigma_c^2}{\sigma_r^2} r_c(\tau)\cos\omega_c\tau + \\ & + \frac{\sigma_m^2}{\sigma_r^2} r_m(\tau)\cos\omega_m\tau + \frac{\sigma_c}{\sigma_r} r_n(\tau + \tau_з)\cos[\omega_0 t - \omega_c\tau - \\ & - \omega_c\tau_з - \zeta_n(\tau + \tau_з)] + \frac{\sigma_c}{\sigma_r} r_n(\tau_з - \tau)\cos[\omega_0 t - \omega_c\tau_з + \\ & + \omega_r\tau - \zeta_n(\tau_з - \tau)], \end{aligned} \tag{4.9}$$

where $\omega_0 = \omega_r - \omega_c$.

Dispersion of the input voltage equals

$$\sigma_{y(t)}^2 = \sigma_r^2 \rho(t,\ 0);\ \sigma_{y(t+\tau)}^2 = \sigma_r^2 \rho(t + \tau,\ 0). \tag{4.10}$$

4.2. REGULAR VOLTAGE COMPONENT AT CORRELATOR OUTPUT

Our problem is to find the regular component of the voltage at the output of a functional correlator in the intermediate frequency range (intermediate frequency ω_0 is equal to the difference in average frequency spectra of the input signal and the reference voltage). Using the transformation method we find the set-averaged value of voltage at the output of the nonlinear element, which also represents the regular component.

Let us find the transition function of the nonlinear element as the unilateral Laplace transform from its characteristic, which can be determined by expression (4.1)

$$f(w)=\int_{0}^{\infty} a y^{\nu} e^{-wy} dy.$$

For nonlinear devices of the ν-th power the Laplace transform exists if Re(w) > 0 and is equal [14] to

$$f(w)=\frac{a\Gamma(\nu+1)}{w^{\nu+1}}, \qquad (4.11)$$

where Γ(x) is the gamma-function.

The response of the nonlinear element can be calculated from the known transition function and the output effect by means of the reverse Laplace transform:

$$z=\frac{1}{2\pi i}\int_{l} f(w) e^{w} dw. \qquad (4.12)$$

The integration contour is a straight line on complex plane w, is parallel to the imaginary axis, and passes the other at a distance at which all of the particular points of the transition function remain to the left of the contour.

The transition function of (4.11) is an analytical function of complex variable w except for the singular point at the origin of the coordinates w = 0. The response of the nonlinear element to the effect in the form of the regular process can be found by substituting y(t) in formula (4.12) and calculating the obtained integral by means of the apparatus for the theory of the functions of the complex variable. In the studied case the effect is the sum of random processes whose relationship is unsteady. To determine the regular component in the response of the nonlinear element it is necessary to study a whole group of occurrences of the random process at the output and then average this set appropriately [formula (1.2)]. Since response z represents the single-valued function of the effect, then by considering (4.12) we will get the following for the regular component of the response of the nonlinear element:

$$\overline{z(t)} = \frac{1}{2\pi i}\int_{l} f(w)\,\overline{\exp[wy(t)]}\,dw.$$

Thus, the regular component of the response is found by statistical averaging of the function from the input effect, which can be done by means of the characteristic function. In the analyzed case the input effect represents the superimposition of the three gaussian processes and, consequently, the distribution of the sum process will be gaussian. For a gaussian random process with a zero average value and with dispersion $\sigma^2_{y(t)}$ the characteristic function will be equal [14] to

$$\overline{\exp[wy(t)]} = \exp\left[\frac{1}{2}\sigma^2_{y(t)}w^2\right],$$

where w = iv.

Then, on the basis of (4.11) the regular component of the response will have the form of

$$\overline{z(t)} = \frac{a\Gamma(\nu+1)}{2\pi i}\int_{l} w^{-\nu-1}\exp\left[\frac{1}{2}\sigma^2_{y(t)}w^2\right]dw. \qquad (4.13)$$

Calculation of the integral (see appendix) with formula (4.10) considered for input voltage dispersion gives us the following expression:

$$\overline{z(t)} = \frac{a\Gamma(\nu+1)\sigma_r^{\nu}}{2^{1+\frac{\nu}{2}}\Gamma\left(1+\frac{\nu}{2}\right)}[\rho(t,0)]^{\frac{\nu}{2}}. \qquad (4.14)$$

Let us make a harmonic analysis of the regular component of the response. From expression (4.9) it follows that when $\tau = 0$

$$\rho(t,0) = \frac{\sigma_n^2}{\sigma_r^2}\left\{1 + 2\frac{\sigma_c\sigma_r}{\sigma_n^2}r_n(\tau_3)\cos[\omega_0 t - \omega_0\tau_3 - \varphi_n(\tau_3)]\right\},$$

where

$$\sigma_n^2 = \sigma_c^2 + \sigma_e^2 + \sigma_m^2.$$

For $[\rho(t, 0)]^{\nu/2}$ we will use a representation in the form of the following series [13, p. 35]:

$$(1+x)^q = 1 + qx + \frac{q(q-1)}{2!}x^2 + \ldots |x| < 1.$$

Then

$$[\rho(t,\ 0)]^{\nu/2} = \left(\frac{\sigma_n}{\sigma_r}\right)^{\nu}\left\{1 + \sum_{k=1}^{\infty}\frac{\frac{\nu}{2}\left(\frac{\nu}{2}-1\right)\ldots\left(\frac{\nu}{2}-k+1\right)}{k!}\times\right.$$
$$\left.\times\left[2\frac{\sigma_c\sigma_r}{\sigma_n^2}r_n(\tau_3)\right]^k \cos^k[\omega_0 t - \omega_c\tau_3 - \zeta_n(\tau_3)]\right\}.$$

Let us examine the regular component of the output voltage in the intermediate frequency range ω_0. The first harmonic of frequency ω_0 in the expression for $[\rho(t, 0)]^{\nu/}$ [sic] will only be for uneven numbers of $k = 2m - 1$. In this case, if we use the expansion [13, p. 40]

$$\cos^{2m-1}x = \frac{1}{2^{2m-2}}\sum_{n=0}^{\infty}\frac{(2m-1)!}{(2m-n-1)!n!}\cos(2m-2n-1)x,$$

Then for the intermediate frequency range we get

$$[\rho(t,\ 0)]_{\omega_0}^{\nu/2} = \left(\frac{\sigma_n}{\sigma_e}\right)^{\nu}\left\{1 + 2\times\right.$$
$$\times\sum_{m=1}^{\infty}\frac{\left(\frac{\nu}{2}\right)\left(\frac{\nu}{2}-1\right)\cdots\left(\frac{\nu}{2}-2m+2\right)}{(m-1)!m!}\times$$
$$\left.\times\left[\frac{\sigma_c\sigma_e}{\sigma_n^2}r_n(\tau_3)\right]^{2m-1}\right\}\cos[\omega_0 t - \omega_c\tau_3 - \zeta_n(\tau_3)].$$

this expression can be represented as the following hypergeometric function:

$$[\rho(t,0)]^{\nu/2}_{\omega_0} = \left(\frac{\sigma_n}{\sigma_r}\right)^{\nu} \left\{1 + \nu \frac{\sigma_c \sigma_r}{\sigma_n^2} r_n(\tau_3) \times \right.$$

$$\left. \times \cos[\omega_0 t - \omega_c \tau_3 - \zeta_n(\tau_3)]\right\} {}_2F_1\left[\frac{2-\nu}{4}; \frac{4-\nu}{4}; 2; \right.$$

$$\left. \frac{4\sigma_c^2 \sigma_r^2}{\sigma_n^4} r_n^2(\tau_3)\right]. \qquad (4.15)$$

The last factor in the form of ${}_2F_1(a; b; c; x)$ represents a hypergeometric function with two parameters in the numerator and one in the denominator, which can be determined by the series [13]

$${}_2F_1(a; b; c; x) = 1 + \frac{ab}{c1!} x + \frac{a(a+1)b(b+1)}{c(c+1)2!} x^2 + \ldots$$

The hypergeometric series is an analytical function of the argument x and converges when $|x| < 1$. Following this premise the powers of the signal and the noise σ_c^2, $\sigma_\text{ш}^2$ are much less than the power of reference voltage $\sigma_\text{г}^2$.

Thus,

$$\sigma_n \approx \sigma_r; \quad \frac{4\sigma_c^2 \sigma_r^2}{\sigma_n^4} r_n(\tau_3) \ll 1.$$

For a low argument value the hypergeometrical function is close to one. Then, the regular component of the response of the nonlinear element at intermediate frequency will equal

$$\overline{z(t)}_{\omega_0} = \frac{a\Gamma(\nu+1)\sigma_r^{\nu}}{2^{1+\frac{\nu}{2}}\Gamma\left(1+\frac{\nu}{2}\right)} \left\{1 + \nu \frac{\sigma_c}{\sigma_r} r_n(\tau_3) \times \right.$$

$$\left. \times \cos[\omega_0 t - \omega_c \tau_3 - \zeta_n(\tau_3)]\right\}. \qquad (4.16)$$

Thus, when the small signal and the reference voltage exert a combined effect on a nonlinear element with a half-wave power characteristic at the output, there appears a voltage component of difference (intermediate) frequency ω_0, whose amplitude is

proportional to the envelope of the cross-correlation function of the reference voltage and the signal.

Let us examine the error which develops when we measure the envelope of the normalized cross-correlation function. From (4.14) and (4.15) we find the expression for the amplitude of the regular component

$$A = \frac{a\Gamma(\nu+1)\, e_n^{\nu-2}}{2^{1+\frac{\nu}{2}}\,\Gamma\left(1+\frac{\nu}{2}\right)}\, e_c e_r^{\nu}\, r_e(\tau_3) \times$$
$$\times\, {}_2F_1\left(\frac{2-\nu}{4};\ \frac{4-\nu}{4};\ 2;\ \frac{4e_c^2 e_r^2}{e_n^4}\, r_e^2(\tau_3)\right).$$

The maximal value of the amplitude A_{max} is obtained at $\tau_3 = 0$, when $r_p = 1$.

For the error characteristic of the method we examine measurement error, determined by the following expression:

$$\Delta = \frac{\dfrac{A(\tau_3)}{A_{max}} - r_e(\tau_3)}{r_e(\tau_3)} = \frac{{}_2F_1\left(\frac{2-\nu}{4};\ \frac{4-\nu}{4};\ 2;\ \frac{4e_r^2 e_r^2}{e_n^4}\, r_e^2(\tau_3)\right)}{{}_2F_1\left(\frac{2-\nu}{4};\ \frac{4-\nu}{4};\ 2;\ \frac{4e_c^2 e_r^2}{e_n^4}\right)} - 1. \qquad (4.17)$$

The absence of tables for the hypergeometric function prevents us from accurately calculating the expected measurement error. In the case of a low input signal we can limit ourselves to the two terms of the hypergeometric series. Then we will get

$$\Delta = \frac{e_c^2}{e_r^2}\left(1-\frac{\nu}{2}\right)\left(1-\frac{\nu}{4}\right)\left[r_e^2(\tau_3)-1\right]. \qquad (4.18)$$

Consequently, measurement error has approximately the same order of smallness as the ratio of powers of the signal and reference voltage. As we might expect, the use of a nonlinear element with a unilateral square-law characteristic does not lead to error in

measuring correlation coefficients when studying gaussian processes. Formula (4.18) does not consider error caused by extraneous noise and by the finiteness of the averaging time.

4.3. AUTOCORRELATION FUNCTION AND ENERGY SPECTRUM OF VOLTAGE AT OUTPUT OF NONLINEAR ELEMENT

At the output of the nonlinear element in a functional correlator averaging occurs over a period of time which is considerably greater than the correlation time of the studied random processes. Averaging is done by means of a band filter, which is placed behind the nonlinear element and tuned to difference frequency ω_0. Since averaging time is finite, along with the regular voltage component (which exists in the presence of a signal) there is always a noise component at the filter output. Consequently, the output voltage of the filter is a random process, and one must have a knowledge of statistics in order to describe it. The nonlinear transformations which occur above the gaussian random processes lead to nongaussian statistics. In the particular case, which has greater practical interest, where the band pass of the averaging filter is much narrower than the bands of the energy spectra of input oscillations, the random process at the filter output "tends toward normalization," i.e., the density of probability distribution in this process is close to normal. For normalization the width of the energy spectra of the input signals must exceed the band pass of the averaging filter. This depends on the degree to which the law of distribution at the input of the averaging filter deviates from the normal. We can assume with sufficient practical accuracy this ratio to be on the order of 6-10 [24].

Let us estimate the effectiveness of a functional correlator from the ratio of the power of the regular component to the power of fluctuations at the output of the averaging filter. We will compare a functional correlator with a correlator in which ideal multiplication occurs.

To solve the problem we must find the correlation function of the response of the nonlinear element to the input effect in the form of the sum of unsteadily related random processes. The unknown correlation function will be time dependent. The energy spectrum of the output voltage of the nonlinear element is found by applying the Wiener-Khinchin transform to the time-averaged autocorrelation function [24].

By means of formula (4.12) the autocorrelation function of the response of the nonlinear element is written in the form of

$$K_z(t, \tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} z(t)\, z(t+\tau)\, p[z(t);\ z(t+\tau)]\, dz(t)\, dz(t+\tau) =$$
$$= \left(\frac{1}{2\pi i}\right)^2 \int_l \int_l \overline{\exp[w_1 y(t) + w_2 y(t+\tau)]}\, f(w_1) f(w_2)\, dw_1 dw_2.$$

Statistical averaging $\overline{\exp[w_1 y(t) + w_2 y(t+\tau)]}$ represents the transformed two-dimensional characteristic function, which for the normal random process equals [14]

$$\overline{\exp[iv_1 y(t) + iv_2 y(t+\tau)]} = \exp[iv_1 \overline{y(t)} +$$
$$+ iv_2 \overline{y(t+\tau)} - \frac{1}{2} v_1^2 \overline{y^2(t)} - \frac{1}{2} v_2^2 \overline{y^2(t+\tau)} -$$
$$- v_1 v_2 \overline{y(t) \cdot y(t+\tau)}].$$

If we assume $w_1 = iv_1$; $w_2 = iv_2$ and consider (4.8) and (4.10), then we get

$$\overline{\exp[w_1 y(t) + w_2 y(t+\tau)]} = \exp\Big[\frac{1}{2} \sigma_y^2 \rho(t, 0) w_1^2 +$$
$$+ \sigma_y^2 \rho(t, \tau) w_1 w_2 + \frac{1}{2} \sigma_y^2 \rho(t+\tau, 0) w_2^2\Big].$$

By expanding the exponential function in a series, we get the following expression for the autocorrelation function of the response of the nonlinear element:

$$K_2(t, \tau) = \sum_{k=0}^{\infty} \frac{\sigma_r^{2k} \rho^k(t, \tau)}{k!} \frac{1}{2\pi i} \int_l w_1^k f(w_1) \times$$
$$\times e^{1/2\sigma_r^2 \rho(t, 0) w_1^2} dw_1 \frac{1}{2\pi i} \int_l w_2^k f(w_2) e^{1/2\sigma_r^2 \rho(t+\tau, 0) w_2^2} dw_2.$$

Here let us substitute the expression for the transitional function of (4.11). Then, by means of contour integral (P. 1.1, appendix) we get

$$K_2(t, \tau) = \frac{a^2 \Gamma^2(\nu + 1)}{2^{\nu+2}} \sigma_r^{2\nu} \sum_{k=0}^{\infty} \frac{2^k}{k! \Gamma^2\left(\frac{2 - k + \nu}{2}\right)} \times$$
$$\times [\rho(t, 0) \rho(t + \tau, 0)]^{(\nu-k)/2} \rho^k(t, \tau). \tag{4.19}$$

Let us calculate the time-averaged autocorrelation function for the response of the nonlinear element. For every number "k" we must average the following product with respect to time:

$$\rho^{(\nu-k)/2}(t, 0) \rho^{(\nu-k)/2}(t + \tau, 0) \rho^k(t, \tau).$$

Averaging is done for the most interesting case, in which σ_c, $\sigma_\omega \ll \sigma_r$. The individual factors of the averaged expression should be written in the form of series

$$[\rho(t, 0)]^{(\nu-k)/2} = 1 + \frac{\left(\frac{\nu-k}{2}\right)}{1!} \xi_1(t) +$$
$$+ \frac{\left(\frac{\nu-k}{2}\right)\left(\frac{\nu-k}{2} - 1\right)}{2!} \xi_1^2(t) + \ldots,$$
$$[\rho(t + \tau, 0)]^{(\nu-k)/2} = 1 + \frac{\left(\frac{\nu-k}{2}\right)}{1!} \xi_2(t) +$$
$$+ \frac{\left(\frac{\nu-k}{2}\right)\left(\frac{\nu-k}{2} - 1\right)}{2!} \xi_2^2(t) + \ldots,$$
$$[\rho(t, \tau)]^k = \eta_1^k(\tau) + \frac{k}{1!} \eta_1^{k-1}(\tau) \xi(t, \tau) +$$
$$+ \frac{k(k-1)}{2!} \eta_1^{k-2}(\tau) \xi^2(t, \tau) + \ldots$$

In multiplying these expressions we average only those terms which have an order of smallness of no less than σ_c^2/σ_r^2, σ_m^2/σ_r^2. Finally we get

$$
\begin{aligned}
<[\rho(t,0)\rho(t+\tau,0)]^{(\nu-k)/2}\rho^k(t,\tau)> &= r_r^k(\tau)\cos^k\omega_r\tau + \\
&+ k\frac{\sigma_c^2}{\sigma_r^2} r_r^{k-1}(\tau)\cos^{k-1}\omega_r\tau\, r_0(\tau)\cos\omega_0\tau + \\
&+ k\frac{\sigma_m^2}{\sigma_r^2} r_r^{k-1}(\tau)\cos^{k-1}\omega_r\tau\, r_m(\tau)\cos\omega_m\tau + \\
&+ k\left(\frac{\nu-k}{2}\right)\frac{\sigma_c^2}{\sigma_r^2} r_r^{k-1}(\tau)\cos^{k-1}\omega_r\tau\, r_0(\tau_3)\times \\
&\times \{r_0(\tau+\tau_3)\cos[\omega_c\tau+\zeta_0(\tau+\tau_3)-\zeta_0(\tau_3)] + \\
&+ r_0(\tau_3-\tau)\cos[\omega_c\tau+\zeta_0(\tau_3)-\zeta_0(\tau_3-\tau)]\} + \\
&+ \frac{k(k-1)}{2!}\frac{\sigma_c^2}{\sigma_r^2} r_r^{k-2}(\tau)\cos^{k-2}\omega_r\tau\, r_0(\tau_3-\tau)\times \\
&\times r_0(\tau_3+\tau)\cos[(\omega_r+\omega_c)\tau+\zeta_0(\tau_3+\tau) - \\
&- \zeta_0(\tau_3-\tau)] + r_r^k(\tau)\cos^k\omega_r\tau\left(\frac{\nu-k}{2}\right)^2\times \\
&\times 2\frac{\sigma_c^2}{\sigma_r^2} r_0^2(\tau_3)\cos\omega_0\tau. \qquad (4.20)
\end{aligned}
$$

Let us assume that intermediate frequency ω_0 is much smaller than the central frequencies of the signal ω_c, reference voltage ω_r, and the combined components in the form of $k\omega_r \pm \omega_c$, $k\omega_r \pm \omega_m$ when $k \geq 2$. Then (from 4.20) it follows that the component of the time-averaged correlation function (4.19) will correspond to the region of low and intermediate frequencies only at even numbers of $k = 2n$. Thus, if we consider the relationship [42, p. 44]

$$\frac{1}{\Gamma(z-n)} = \frac{(z-1)(z-2)\ldots(z-n)}{\Gamma(z)}$$

and use the expressions for powers of trigonometric functions via the multiple argument function [13, p. 39]

$$\cos^{2n} x = \frac{1}{2^{2n}}\left\{\sum_{k=0}^{n-1}\frac{2n!}{(2n-k)!\,k!}\cos 2(n-k)x + \frac{2n!}{n!\,n!}\right\},$$

$$\cos^{2n-1}x = \frac{1}{2^{2n-2}} \sum_{k=0}^{n-1} \frac{(2n-1)!}{(2n-k-1)!\,k!} \cos(2n-2k-1)x, .$$

then we get the following expressions for the time-averaged correlation function of the response of the nonlinear element corresponding to the region of intermediate and low frequencies:

$$R_{s}^{\text{нч}}(\tau) = R_{oз} + R_{oш} + R_{oc}.$$

we have

$$R_{oз} = D(\nu)\left\{1 + \sum_{n=1}^{\infty} C(\nu,\ n)\, r_{r}^{2n}(\tau)\right\}; \qquad (4.21$$

$$R_{oш} = D(\nu)\frac{2\sigma_{ш}^{2}}{\sigma_{r}^{2}}\cos(\omega_{r}-\omega_{ш})\tau \sum_{n=1}^{\infty} nC(\nu,\ n)\, r_{r}^{2n-1}(\tau)\, r_{ш}(\tau); \qquad (4.22)$$

$$R_{oc} = D(\nu)\frac{2\sigma_{c}^{2}}{\sigma_{r}^{2}}\left\{\frac{\nu^{2}}{4} r_{в}^{2}(\tau_{з})\cos\omega_{0}\tau + \sum_{n=1}^{\infty} nC(\nu,\ n)\, r_{r}^{2n-2}(\tau)\,\times\right.$$
$$\times\left\{\cos\omega_{0}\tau\left[\frac{\left(\frac{\nu}{2}-n\right)^{2}}{n} r_{r}^{2}(\tau)\, r_{в}^{2}(\tau_{з}) + r_{r}(\tau)\, r_{c}(\tau)\right] +\right.$$
$$+\left(\frac{\nu}{2}-n\right) r_{в}(\tau_{з})\, r_{r}(\tau)\Big[\Big[r_{в}(\tau+\tau_{з})\cos[\omega_{0}\tau - \zeta_{в}(\tau+\tau_{з}) +$$
$$+ \zeta_{в}(\tau_{з})] + r_{в}(\tau_{з}-\tau)\cos[\omega_{0}\tau + \zeta_{в}(\tau_{з}-\tau) - \zeta_{в}(\tau_{з})]\Big]\Big] +$$
$$+(n-1)\, r_{в}(\tau_{з}-\tau)\, r_{в}(\tau_{з}+\tau)\cos[\omega_{0}\tau -$$
$$\left.\left. - \zeta_{в}(\tau+\tau_{з}) + \zeta_{в}(\tau_{з}-\tau)]\right\}\right\}; \qquad (4.23)$$

$$D(\nu) = \frac{a^{2}\Gamma^{2}(\nu)\,\sigma_{r}^{2\nu}}{2^{\nu}\Gamma^{4}\left(\frac{\nu}{2}\right)}, \qquad (4.24)$$

$$C(\nu,\ n) = \frac{\left(\frac{\nu}{2}\right)^{2}\left(\frac{\nu}{2}-1\right)^{2}\cdots\left(\frac{\nu}{2}-n+1\right)^{2}}{(n!)^{2}}. \qquad (4.25)$$

Thus, the time-averaged correlation function for the response of the nonlinear element in the low frequency region consists of three components:

1) $R_{од}$ - correlation function of detected reference voltage (terms of series (4.21) contain factors in the form of $r_{г}^{2n}(\tau)$, which describe reference voltage only),

2) $R_{ош}$ - correlation function of extraneous noise transformed by reference voltage [terms of series (4.22) have factors in the form of $r_{г}^{2n-1}(\tau)$, $r_{ш}(\tau)$],

3) $R_{ос}$ - time-averaged correlation function of signals, transformed by reference voltage (series (4.23), consists of terms containing parameters of cross-correlation and autocorrelation functions of signal and reference voltage).

From (4.23) it follows that the time-averaged correlation function of the signal transformed by the reference voltage and also the energy spectrum of the self-noise of the functional correlator are time delay functions of the signal relative to the reference voltage.

In estimating the effectiveness of a functional correlator we limit ourselves to the simpler case, where

$$\tau_з = 0,\ \zeta_в(\tau) \equiv 0,\ \omega_ш = \omega_c,\ r_г(\tau) = r_c(\tau) = r_ш(\tau) = r(\tau). \qquad (4.26)$$

The assumptions of (4.26) coincide with the limits of (2.5) used earlier in analyzing an ideal correlator.

This last equality indicates that the energy spectra of the signal and reference voltage differ only in their central frequencies. Under these conditions the application of the direct Wiener-Khinchin transform to (4.21), (4.22), and (4.23) gives us the following expressions for corresponding energy spectra:

$$S_{o\text{д}} = D(\nu)\left\{\delta(f) + \sum_{n=1}^{\infty} C(\nu,\ n)\,{}_{2n}S_c(f)\right\}. \tag{4.27}$$

$$S_{o\text{ш}} = D(\nu)\frac{\sigma_{\text{ш}}^2}{\sigma_r^2}\sum_{n=1}^{\infty} nC(\nu,\ n)\left[{}_{2n}S_{r\text{ш}}(f-f_o) + {}_{2n}S_{r\text{ш}}(f+f_o)\right]; \tag{4.28}$$

$$S_{oc} = D(\nu)\frac{\sigma_c^2}{\sigma_r^2}\left(\frac{\nu}{2}\right)^2\left\{\delta(f-f_o) + \delta(f+f_o) + \right.$$

$$\left. + \sum_{n=1}^{\infty} C(\nu,\ n)\left[{}_{2n}S_c(f-f_o) + {}_{2n}S_c(f+f_o)\right]\right\}. \tag{4.29}$$

where $\delta(f)$ is the Dirac delta-function;

$${}_{2n}S_c(f) = \int_{-\infty}^{\infty} r^{2n}(\tau)\,e^{-i2\pi f\tau}d\tau;$$

$${}_{2n}S_{r\text{ш}}(f) = \int_{-\infty}^{\infty} r^{2n-1}(\tau)\,r_{\text{ш}}(\tau)\,e^{-i2\pi f\tau}d\tau.$$

We can find the signal/noise ratio at the functional correlator output by using the expression obtained.

4.4. SIGNAL/NOISE RATIO AT CORRELATOR OUTPUT

Our problem is to compare the signal/noise ratio at the output of a functional correlator with that of an ideal correlator with other conditions remaining equal. Let us find the signal/noise ratio at the output of a functional correlator. The reasoning and transformation will be absolutely analogous to those presented in § 2.2 in deriving (2.7). Based on relationships (4.27)-(4.29) the signal/noise ratio at the output of a functional correlator will be

$$q_{\phi\text{ш}} = \frac{P_c}{P_{\text{ш}}^{c\text{б}} + P_{\text{ш}}^{c\text{т}}} = \frac{2}{B_\nu}\Bigg\{\sum_{n=1}^{\infty} C(\nu,\ n)\,[_{2n}S_c(0) +$$

$$+ {}_{2n}S_c(2f_0)] + \left(\frac{2}{\nu}\right)^2 \frac{\sigma_r^2}{\sigma_c^2} \sum_{n=1}^{\infty} C(\nu,\ n)\cdot {}_nS_c(f_0) +$$

$$+ \left(\frac{2}{\nu}\right)^2 \frac{\sigma_{\text{ш}}^2}{\sigma_c^2} \sum_{n=1}^{\infty} nC(\nu,\ n)\,[_{2n}S_{r\text{ш}}(0) +$$

$$+ {}_{2n}S_{r\text{ш}}(2f_0)]\Bigg\}^{-1}. \qquad (4.30)$$

This expression cannot be conveniently used in this form, since the degree of superiority of the ideal correlator over the functional correlator with the nonlinear element is not apparent. Expression (4.30) will be written as

$$q_{\phi\text{ш}} = \frac{2}{B_\nu}\left\{\lambda\left(1 + \mu\frac{\sigma_r^2}{\sigma_c^2}\right)[_2S_c(0) + {}_2S_c(2f_0)] + \right.$$

$$\left. + \beta\frac{\sigma_{\text{ш}}^2}{\sigma_c^2}\,[_2S_{r\text{ш}}(0) + {}_2S_{r\text{ш}}(2f_0)]\right\}^{-1}, \qquad (4.31)$$

where we designate:

$$\mu = \frac{\left(\frac{2}{\nu}\right)^2 \sum_{n=1}^{\infty} C(\nu,\ n)\cdot {}_{2n}S_c(f_0)}{\sum_{n=1}^{\infty} C(\nu,\ n)\,[_{2n}S_c(0) + {}_{2n}S_c(2f_0)]}; \qquad (4.32)$$

$$\lambda = \sum_{n=1}^{\infty} C(\nu,\ n)\,\frac{_{2n}S_c(0) + {}_{2n}S_c(2f_0)}{_2S_c(0) + {}_2S_c(2f_0)}, \qquad (4.33)$$

$$\beta = \left(\frac{2}{\nu}\right)^2 \sum_{n=1}^{\infty} nC(\nu,\ n)\,\frac{_{2n}S_{r\text{ш}}(0) + {}_{2n}S_{r\text{ш}}(2f_0)}{_2S_{r\text{ш}}(0) + {}_2S_{r\text{ш}}(2f_0)}. \qquad (4.34)$$

If we compare the signal/noise ratio at the output of an ideal correlator [formula (2.7)] with the same ratio for a functional correlator [formulas (4.30) or (4.31)] we will be able to give the following physical interpretation to the reduced loss coefficient:

$\mu\frac{e_r^2}{e_c^2}$ shows how many times noise at the output of the averaging filter, which is caused by the detected reference voltage, exceeds noise caused by the random nature of the signal;

λ indicates the factor by which the signal/noise ratio at the output of a functional correlator is inferior to that of an ideal correlator in a case where the noise of the detected reference voltage (which may occur when a balanced system is used) and extraneous noise are absent;

β shows the factor by which the signal/noise ratio at the output of a functional correlator is inferior to that of an ideal correlator in a case where only extraneous noise transformed by the reference voltage is considered (in practical terms this occurs when the spectral density of extraneous noise is much greater than the spectral density of self-noise and detected noise).

Let us examine in more detail the coefficient of losses β for two cases:

1. The energy spectrum of extraneous noise is much wider than the reference voltage spectrum.

2. The energy spectrum of extraneous noise is the same as the signal spectrum.

<u>Case 1</u>. In order to calculate the values of the coefficient of losses β we must know the spectral density in the form of ${}_{2n}S_{r\text{ш}}(f)$. When n = 1 we have

$${}_{2}S_{r\text{ш}}(f)=\int_{-\infty}^{\infty} r_{\text{ш}}(\tau)\,r(\tau)\,e^{-i2\pi f\tau}\,d\tau.$$

For the studied case the correlation function $r_{\text{ш}}(\tau)$ is much narrower than $r(\tau)$. Thus, we can write approximately

$$ {}_2S_{r\text{ш}}(f) \approx \int_{-\infty}^{\infty} r_{\text{ш}}(\tau)\, r(0)\, e^{-i2\pi f\tau}\, d\tau = S^{\text{н}}_{\text{ш}}(f). $$

Here the energy spectrum $S^{\text{н}}_{\text{ш}}(f)$ corresponding to correlation function $r_{\text{ш}}(\tau)$ is the energy spectrum of extraneous noise which has been transferred from frequency $\omega_{\text{ш}}$ to "zero" frequency. It is quite obvious that the approximation ${}_{2n}S_{r\text{ш}}(f) \approx S^{\text{н}}_{\text{ш}}(f)$ will continue to decrease in accuracy as number n increases, since in this case the correlation function $r^{2n-1}(\tau)$ continues to narrow. However, as we will show below, series (4.34) in view of its rapid convergence is practically determined by the first terms alone. If we consider this peculiarity, we assume that

$$ {}_{2n}S_{r\text{ш}}(f) = \int_{-\infty}^{\infty} r_{\text{ш}}(\tau)\, r^{2n-1}(\tau)\, e^{-i2\pi f\tau}\, d\tau \approx S^{\text{н}}_{\text{ш}}(f). $$

Then the expression for the coefficient of losses β with the coefficients of (4.25) considered will acquire the form of

$$ \beta = 1 + \sum_{n=1}^{\infty} \frac{\left(1-\frac{\nu}{2}\right)^2 \left(2-\frac{\nu}{2}\right)^2 \cdots \left(n-\frac{\nu}{2}\right)^2}{n!\,(n+1)!}. \qquad (4.35) $$

Using a particular value of the hypergeometric function [27, p. 709]

$$ {}_2F_1(\alpha,\beta,\gamma,1) = 1 + \frac{\alpha\beta}{\gamma\cdot 1!} + \frac{\alpha(\alpha+1)\cdot\beta\cdot(\beta+1)}{\gamma(\gamma+1)\cdot 2!} + \ldots = \frac{\Gamma(\gamma)\cdot\Gamma(\gamma-\alpha-\beta)}{\Gamma(\gamma-\alpha)\,\Gamma(\gamma-\beta)}, $$

we represent the coefficient of losses in the form of

$$ \beta = \frac{\Gamma(\nu)}{\Gamma^2\left(1+\frac{\nu}{2}\right)}. \qquad (4.36) $$

Case 2. The equality of the width of the noise energy spectrum to the width of the signal energy spectrum means that

$$r_{m}(\tau)=r(\tau);\ {}_{2n}S_{rm}(f)={}_{2n}S_{c}(f).$$

In this case formula (4.34) acquires the form of

$$\beta=\sum_{n=1}^{\infty}\frac{4n}{\nu^{2}}C(\nu,\ n)\frac{{}_{2n}S_{c}(0)+{}_{2n}S_{c}(2f_{0})}{{}_{2}S_{c}(0)+{}_{2}S_{c}(2f_{0})}. \qquad (4.37)$$

From relationships (4.32)-(4.37) it follows that the loss coefficients μ, λ, β depend on the characteristic of the nonlinear element (parameter ν) and the shape of the energy spectrum of oscillations acting on the functional correlator.

4.5. SPECTRAL DENSITY CHARACTERISTICS OF RANDOM PROCESSES (PARTICULAR CASES)

Loss coefficients are expressed as functions in the form of ${}_{2n}S_{c}(f)$, which depend on the shape of the energy spectrum of the studied random processes. Let us find these functions for several random processes obtained, for example, when "white" noise passes through specific shaping filters.

1. Ideal Filters

Let us describe the energy spectrum by the following function:

$$S_{c}^{\bullet\bullet}(f)=\begin{cases}\frac{\sigma_c^2}{2\Delta f} & \text{при } f_c-\frac{\Delta f}{2}\leqslant|f|\leqslant f_c+\frac{\Delta f}{2},\\ 0 & \text{at other values of } f,\end{cases}$$

[при = when]

where Δf is the width of the energy spectrum.

We know [24, 37] that the correlation function of a process with this spectral density has the form of

$$R_{c}(\tau)=\frac{\sigma_c^2}{\pi\Delta f\tau}\sin\pi\Delta f\tau\cos\omega_c\tau.$$

The energy spectrum corresponding to correlation coefficient $r(\tau)$ is determined by the relationship

$$S_c(f)=\begin{cases}\frac{1}{\Delta f} & \text{при } |f|<\frac{\Delta f}{2},\\ 0 & \text{at other values of } f.\end{cases} \tag{4.38}$$

[при = when]

Let us find spectral density ${}_{2n}S_c(f)$ by means of a convolution type integral [14]:

$$\begin{aligned}{}_2S_c(f)&=\int_{-\infty}^{\infty} r^2(\tau)\, e^{-i2\pi f\tau}\, d\tau=\\ &=\int_{-\infty}^{\infty} S_c(z)\, S_c(f-z)\, dz.\end{aligned}$$

After we have calculated the convolution integral for a rectangular energy spectrum, we get

$${}_2S_c(x)=\begin{cases}\frac{1}{\Delta f}(1-|x|) & \text{при } |x|\leq 1,\\ 0 & \text{at other values of } x,\end{cases}$$

where $x = f/\Delta f$ is normalized frequency.

Similar calculations for convolutions when $n = 2$ give us

$${}_4S_c(x)=\begin{cases}\frac{1}{\Delta f}\left(0{,}5\,|x|^3-|x|^2+\frac{2}{3}\right) & \text{при } 0\leq|x|\leq 1,\\ \frac{1}{\Delta f}\left(-\frac{1}{6}|x|^3+|x|^2-2|x|+\frac{4}{3}\right) & \text{при } 1\leq|x|\leq 2,\\ & \text{at other values of } x.\end{cases}$$

Finding convolutions of a higher order involves complex and tiresome calculations. For this reason we will use approximate methods. Let us use the central limiting theorem of the theory of probabilities. We know [14] that the density of probability distribution of the sum of independent random quantities is determined by the convolution of the probability distribution

density of the terms. If the random quantities fulfill the Lindeberg condition [11, p. 257], then the density of probability distribution for the sum of a great number of independent random quantities will approach the gaussian distribution. The Lindeberg condition is the singular requirement that random quantities have deviations of uniform smallness from their average value, i.e., that the [illegible] of a random quantity be finite. Let us examine relation [illegible].38) as the density of probability distribution of random [illegible] f. In this case ${}_{2n}S_c(f)$ is the density of probability distribution of the sum 2n of identical independent random quantities. The distribution dispersion of (4.38) will be [11, p. 173]

$$\sigma_0^2 = \frac{(\Delta f)^2}{12},$$

while the average value is equal to zero. Since the dispersion of sum 2n of independent random quantities equals

$$\sigma_{2n}^2 = 2n\sigma_0^2,$$

then for ${}_{2n}S_c(x)$ we get the approximate formula

$${}_{2n}S_c(x) = \begin{cases} \dfrac{1}{\sqrt{2\pi}\,\Delta f}\sqrt{\dfrac{6}{n}}\, e^{-\frac{1}{2}\left(\sqrt{\frac{6}{n}}\,x\right)^2} & \text{при } |x| < n, \\ [illegible] & \text{at other values of } x. \end{cases} \qquad (4.39)$$

Table 4.1 shows the values of the convolutions as a function of normalized frequency x. For ${}_4S_c(x)\Delta f$ and ${}_6S_c(x)\Delta f$ in Table 4.1 two values are given. The upper row of values was calculated by approximate formula (4.39). Coincidence is good even for a small n number.

Table 4.1.

x	0	0,5	1,0	1,5	2,0	2,5	3,0	4,0
${}_2S_c(x)\Delta f$	1	0,5	0	0	0	0	0	0
${}_4S_c(x)\Delta f$	0,684 0,[illegible]	0,471 0,47[illegible]	0,158 0,10[illegible]	0,0256 0,021	0,00205 0	0 0	0 0	0 0
${}_6S_c(x)\Delta f$	0,565 0,55	0,44 0,438	0,2[illegible]7 0,217	0,0547 0,054	0,0143 0,00[illegible]44	$6,94\cdot10^{-[illegible]}$ 0	0 0	0 0
${}_8S_c(x)\Delta f$	0,494	0,4[illegible]	0,23	0,0[illegible]	0,025	0,00361	0	0
${}_{10}S_c(x)\Delta f$	0,436	0,376	0,261	0,113	0,03[illegible]	0,0197	$3\cdot10^{-[illegible]}$	0

2. Single Oscillatory Circuit

In thi[illegible] energy spectrum is determined on the following de[illegible]:

$$S_c^{\mathrm{sr}}(f)=\frac{\sigma_c^2}{\pi\Delta f}\left\{\frac{1}{1+\left[\frac{2(f-f_c)}{\Delta f}\right]^2}+\right.$$
$$\left.+\frac{1}{1+\left[\frac{2(f+f_c)}{\Delta f}\right]^2}\right\},$$

where Δf is the width of the energy spectrum on a level of -3 dB. The energy spectrum which corresponds to the correlation coefficient $r(\tau)$ is determined by the expression

$$S_c(f)=\frac{2}{\pi\Delta f}\cdot\frac{1}{1+\left(\frac{2f}{\Delta f}\right)^2}. \qquad (4.40)$$

In this case ${}_{2n}S_c(f)$ can be more conveniently calculated by means of the direct Wiener-Khinchin transform, rather than through the convolution. If we use the theory of remainders in calculating the correlation function corresponding to the energy spectrum of (4.40), we get

$$r(\tau)=\int_{-\infty}^{\infty}S_c(f)\,e^{i2\pi f\tau}\,df=e^{-\pi\Delta f|\tau|}.$$

Then

$$ {}_{2n}S_c(f) = \int_{-\infty}^{\infty} r^{2n}(\tau) e^{-i2\pi f\tau} d\tau = $$
$$ = \int_{-\infty}^{\infty} e^{-2\pi n\Delta f|\tau| - i2\pi f\tau} d\tau = \frac{1}{\pi n\Delta f\left[1+\left(\frac{f}{n\Delta f}\right)^2\right]}. $$

It is interesting to note that in this case quantity ${}_{2n}S_c(f)$ does not approach the gaussian curve at any value of n. This is because the probability distribution density of (4.40) is the Cauchy distribution density, which has infinite dispersion and, consequently, does not satisfy the Lindeberg condition.

3. Band-Pass Filter Consisting of Two Identical Connected Circuits

The autocorrelation function of a signal shaped by a band-pass filter consisting of two identical connected circuits which have a critical bond between them is determined by expression [72]

$$ R(\tau) = \sigma_0^2 e^{-\frac{\pi|\tau|\Delta f}{\sqrt{2}}} \cos 2\pi f_0\tau \left[\cos\left(\frac{\pi\tau\Delta f}{\sqrt{2}}\right) + \right. $$
$$ \left. + \sin\left(\frac{\pi|\tau|\Delta f}{\sqrt{2}}\right)\right], $$

where Δf is the width of the energy spectrum on a level of -3 dB. The envelope of the autocorrelation function of the signal and the energy spectrum corresponding to it will have the form of

$$ r(\tau) = e^{-\frac{\pi|\tau|\Delta f}{\sqrt{2}}} \left[\cos\left(\frac{\pi\tau\Delta f}{\sqrt{2}}\right) + \sin\left(\frac{\pi|\tau|\Delta f}{\sqrt{2}}\right)\right], $$
$$ S_c(f) = \frac{2\sqrt{2}}{\pi\Delta f\left[1+\left(\frac{2f}{\Delta f}\right)^4\right]}. \qquad (4.41) $$

When calculating quantities ${}_{2n}S_c(f)$ we use the Wiener-Khinchin transform to the power of the autocorrelation function of the signal $r^{2n}(\tau)$. The obtained integrals are calculated by means of relationships [13, p. 491]:

$$\int_0^\infty e^{-px} \cos qx\,dx = \frac{p}{p^2+q^2},$$

$$\int_0^\infty e^{-px} \sin qx\,dx = \frac{q}{p^2+q^2}.$$

From the calculations we get:

$${}_2S_c(x) = \frac{\sqrt{2}(x^2+3)}{4\pi\Delta f\,(x^2+0{,}5)(x^4+1)},$$

$${}_4S_c(x) = \frac{\sqrt{2}}{2\pi\Delta f}\left\{\frac{3}{2+x^2} - \frac{4+x^2}{(4+x^2)^2-8x^2} + \right.$$

$$\left. + \frac{5-2x^2}{(2{,}5+x^2)^2-2x^2}\right\},$$

where $x = f/\Delta f$ is normalized frequency.

The reduced expressions show that the complexity of functions ${}_{2n}S_c(x)$ rapidly increases as number n increases. For calculating ${}_{2n}S_c(x)$ at large values of n in this case we can use the method developed in part 1 of this section. The dispersion of distribution (4.41) equals

$$\sigma_0^2 = \frac{2\sqrt{2}}{\pi\Delta f}\int_{-\infty}^{\infty} \frac{f^2 df}{1+\left(\frac{2f}{\Delta f}\right)^4} = \left(\frac{\Delta f}{2}\right)^2.$$

Then, the approximating gaussian curve for ${}_{2n}S_c(x)$ will be

$${}_{2n}S_c(x) = \frac{1}{\sqrt{2\pi}\,\Delta f}\sqrt{\frac{2}{n}}\,e^{-\frac{1}{2}\left(\sqrt{\frac{2}{n}}\,x\right)^2}. \qquad (4.42)$$

Table 4.2.

x	0	0,25	0,5	0,75	1.0	1.5	2.0	3.0	4.0
${}_2S_c(x)\Delta f$	0,676	0,604	0,459	0,287	0,15	0,0124	0,0103	0,00173	0.00605
$S_c(x)\Delta f$	0,4 0,461	0,386 0,432	0,352 0,384	0,3011 0,306	0,212 0,225	0,1295 0,0962	0,0504 0,0347	0,044 0,00562	0,0001 0,00135
$S(x)\Delta f$	0,326 0,37	0,322 0,359	0,3 0,3295	0,272 0,284	0,232 0,232	0,155 0,135	0 0861 0,065	0,0161 0,0104	0,00161 0,00282
$S_c(x)\Delta f$	0,2825 0,316	0,278 0,306	0,264 0,29	0,245 0,207	0,22 0,226	0,11 0,1184	0,103 0,0866	0,0298 0,0225	0,0053 0 0051

Table 4.2 gives values of ${}_{2n}S_c(x)\Delta f$ at different normalized frequencies x. The upper rows of dual values for n = 2, n = 3, n = 4 correspond to those obtained from the approximate formula (4.42).

From the table it follows that in this case the approximation is rough, particularly at high values of normalized intermediate frequency x, which are the values of most interest to us.

4.6. EFFECTIVENESS OF FUNCTIONAL CORRELATOR

1. Loss Coefficients

In order to estimate the effectiveness of a functional correlator we must examine loss coefficients. Loss coefficient λ is determined by relationship (4.33) in the form of an infinite series. At particular values of ν = 2, 4, 6, ... the series can be easily calculated, since it breaks. In the general case the expression for the sum of the series in a closed form is not found, and thus in the calculations we limit ourselves to a finite number of terms. In calculations and in estimating possible error it is convenient to use an additional series in the following form:

$$\lambda_{np} = \sum_{n=1}^{\infty} C(\nu,\ n) = \sum_{n=1}^{\infty} \frac{\left(\frac{\nu}{2}\right)^2 \left(1-\frac{\nu}{2}\right)^2 \cdots \left(n-1-\frac{\nu}{2}\right)^2}{(n!)^2}.$$

Let us represent the auxiliary series in a closed form by means of gamma-functions [as done in deriving relationship (4.36)]. As a result we get

$$\lambda_{np} = \frac{4\Gamma(\nu)}{\nu\Gamma^2\left(\frac{\nu}{2}\right)} - 1.$$

Calculations for the values of spectral power densities in the form of ${}_{2n}S_c(f)$, conducted in the preceding sections, indicate:

a) at any values of n, f quantity ${}_{2n}S_c \leq 1$;

b) values of ${}_{2n}S_c(f)$ diminish very rapidly as frequency f increases. Thus

$$\frac{{}_{2n}S_c(0) + {}_{2n}S_c(2f_0)}{{}_{2}S_c(0) + {}_{2}S_c(2f_0)} \leqslant 1,$$

where equality occurs only when n = 1. Consequently, the terms of the series (4.33) are smaller than the corresponding terms of the auxiliary series. Based on the criterion of comparing series with positive terms [9], it can be confirmed that the sum of series (4.33) is always less than the sum of the auxiliary series, i.e., $\lambda \leq \lambda_{np}$. When a functional correlator with a nonlinear element is given practical application an intermediate frequency f_0 which is considerably greater than the width of the energy spectrum of the reference voltage must be selected. In this case

$${}_{2n}S_c(2f_0) \ll {}_{2}S_c(0)$$

while relationship (4.33) will have the form of

$$\lambda = \sum_{n=1}^{\infty} \frac{\left(\frac{\nu}{2}\right)^2 \left(1-\frac{\nu}{2}\right)^2 \cdots \left(n-1-\frac{\nu}{2}\right)^2}{(n!)^2} \cdot \frac{{}_{1n}S_0(0)}{{}_1S_0(0)}. \qquad (4.43)$$

Indirect calculations show that when f_0 is used in a frequency range of $0 < f_0 < 4\Delta f$, the change in coefficient λ is extremely insignificant (less than 5%), unless for practical purposes we can assume that the loss coefficient λ does not depend on the intermediate frequency value. The values of loss coefficient λ calculated from formula (4.43) with the first four terms of the series considered are shown in Table 4.3 along with the limiting values of the sum of this series λ_{np} for the three studied energy spectra.

Table 4.3.

Spectrum	Value ν					
	1	1.5	2.0	2.5	3.0	4.0
Rectangular	0,264	0,569	1,0	1,579	2,34	4,666
Band-pass filter	0,263	0,569	1,0	1,58	2,35	4,682
Oscillatory circuit	0,259	0,568	1,0	1,576	2,322	4,5
λ_{np}	0,27	0,573	1,0	1,586	2,393	5,0

From Table 4.3 it follows that:

1) for practical purposes it is sufficient to calculate the sum of the four terms of the series (4.43) which determines loss coefficient λ;

2) coefficient λ is virtually independent of the form of the energy spectrum in the signal (reference voltage) and thus in studying the different random processes it is best to use values calculated for a band-pass filter (the curve shown in Fig. 4.1);

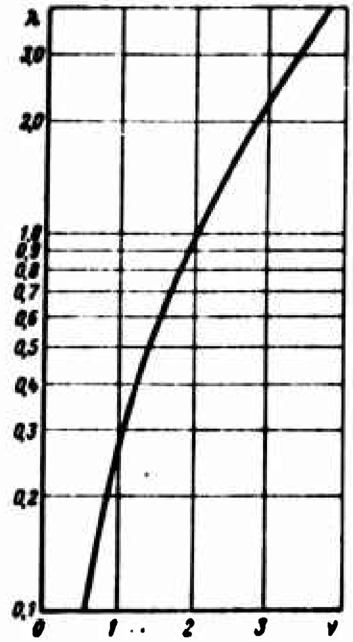

Fig. 4.1. Loss coefficient λ as a function of the characteristic of a nonlinear unit.

3) there exists a substantial dependence between λ and the value of ν, i.e., between λ and the characteristic of the nonlinear device.

It is an interesting fact that when $\nu < 2$ coefficient λ becomes less than one, i.e., the output signal/noise ratio of the functional correlator can be better than that of an ideal correlator. As follows from Fig. 4.1, when $\nu = 1$ this improvement may constitute $(0.259)^{-1} = 3.86$ (or 5.8 dB). This fact contradicts the conclusions of statistical radio engineering, since loss coefficient λ describes only the self-noise of a functional correlator which is calculated when there is a total correlation between the reference voltage and the signal.

The following experiment can be performed to illustrate at a glance the effective reducing self-noise. Let there be two random processes differing only in scale, i.e., the cross-correlation function between the processes is equal to $R = +1$. Let us assume that each of the processes passes through a "rigid" limiter (case $\nu = 0$), so that the voltage at the output of any one of the limiters can acquire only the two values of $+A$ and $-A$, while the transmission from one level to another occurs as a jump. If the outputs of the limiters are connected to the inputs of an ideal correlator, then voltage at the multiplier output will have only the one value of $+A^2$, since in the case of a cross-correlation coefficient equal to 1 there is a simultaneous shift in the responses of the limiters from one level to the other. Consequently,

self-noise will be absent in an ideal correlator. If we assume that the unknown random processes can be time-shifted relative to one another, then at the multiplier output there will be a variable component in addition to the constant component. This results from the fact that the responses of the limiters will have opposite polarities on the individual time segments.

Now let us examine the processes which occur in a functional correlator when $\nu = 1$, i.e., when the functional correlator can be regarded as a "linear" detector. If the power of the reference voltage is much greater than the power of the signal, then the nonlinear element can be regarded in relation to the small signal as linear with time-variable parameters. Here, if $u_r(t) < 0$, then the signal will not pass through the nonlinear element (conductivity is absent); when $u_r(t) > 0$ the signal passes without distortion (conductivity is constant and is determined by the derived characteristic of the nonlinear element at positive values of the input effect). Thus, in the given case passage of the signal and reference voltage through the nonlinear element can be represented as a process of multiplying an undistorted input signal by an amplitude-limited reference voltage. If there is no signal delay relative to the reference voltage, then at the output of the functional correlator there will be self-noise, caused by the random nature of the input signal envelope. Obviously the self-noise in this case should be less than in the case of an ideal correlator, in which the supplied oscillations are not limited. Thus, it follows from our examination that an increase in signal delay relative to reference voltage can lead to an increase in the self-noise of a functional correlator, which has been completely confirmed experimentally.

Study of loss coefficient β, which is determined when the energy spectra of the reference voltage are identical, the signal, and extraneous noise by means of relationship (4.37) is analogous to the study of coefficient λ. With the same reasoning used to

derive relationship (4.43) when $f_0 \gg \Delta f$, we get the following series for β:

$$\beta = 1 + \sum_{m=1}^{\infty} \frac{\left(1-\frac{\nu}{2}\right)^2 \left(2-\frac{\nu}{2}\right)^2 \cdots \left(m-\frac{\nu}{2}\right)^2}{m!\,(m+1)!} \times$$

$$\times \frac{{}_{2m+2}S_c(0)}{{}_2S_c(0)}. \qquad (4.44)$$

For the energy spectrum, as at the output of the single oscillatory circuit, we have

$$\frac{{}_{2m+2}S_c(0)}{{}_2S_c(0)} = \frac{1}{m+1},$$

while the value of the loss coefficient is equal to

$$\beta = 1 + \sum_{m=1}^{\infty} \frac{\left(1-\frac{\nu}{2}\right)^2 \cdot \left(2-\frac{\nu}{2}\right)^2 \cdots \left(m-\frac{\nu}{2}\right)^2}{[(m+1)!]^2} =$$

$$= \frac{4}{\nu^2}\left[\frac{\Gamma(\nu+1)}{\Gamma^2\left(1+\frac{\nu}{2}\right)} - 1\right].$$

The limiting value of expression (4.44) is determined by the sum of the series in majorized terms

$$\beta_{\text{пр}} = 1 + \sum_{m=1}^{\infty} \frac{\left(1-\frac{\nu}{2}\right)^2 \left(2-\frac{\nu}{2}\right)^2 \cdots \left(m-\frac{\nu}{2}\right)^2}{m!\,(m+1)!} =$$

$$= \frac{\Gamma(\nu)}{\Gamma^2\left(1+\frac{\nu_0}{2}\right)}.$$

Thus, the limiting value for loss coefficient β determined by formula (4.44) coincides with the value of the loss coefficient found from formula (4.35) for the case where the width of the energy spectrum of extraneous noise is much greater than the width of the energy spectrum of the signal (reference voltage). Figure 4.2

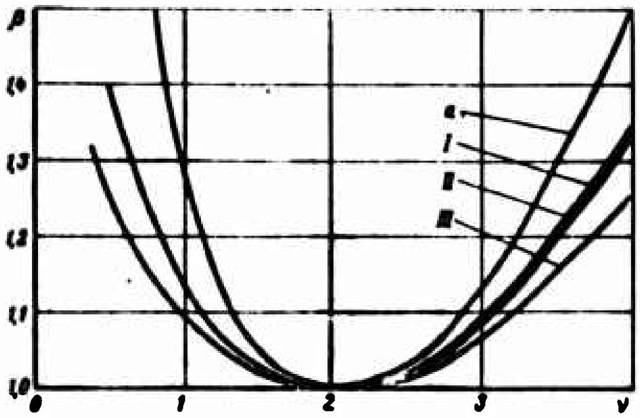

Fig. 4.2. Loss coefficient β as a function of characteristic of nonlinear unit: a - wide-band extraneous noise (limiting value); I - signal spectrum in the form of frequency of band-pass filter characteristic; II - rectangular signal spectrum; III - signal spectrum in the form of frequency characteristic of oscillating circuits.

(curve a) represents the graphic dependence of the limiting value of loss coefficient β on wide-band extraneous noise. In the same figure we see the values of loss coefficient β for the three studied energy spectra of the signals. From these graphic dependences it follows that the loss coefficient relative to extraneous noise is greater than one in all cases. This is found in agreement with the conclusions of statistical radio technology: no functional correlator can separate a useful signal against a background of extraneous noise better than an ideal correlator. The magnitude of loss coefficient β for $1 \leq \nu \leq 3$ does not exceed 1.275, i.e., losses in the signal/noise ratio relative to extraneous noise for a functional correlator are not great compared to those of an ideal correlator. When $\nu > 3$ and when $\nu < 1$ loss coefficient β rises sharply. Loss coefficient β is most interesting from the standpoint of the theory of detection and measurement of signal parameters, since it is extraneous noise that limits maximal sensitivity and accuracy.

In functional correlators an intermediate frequency is selected which is much wider than the energy spectrum of the signal. Thus, the values of loss coefficient ν are of practical interest only in a case where $f_0 > 4\Delta f$. Here the expression for the coefficient of losses μ according to formula (4.32) can be represented with (4.33) considered in a simplified form with an error of less than 1.5%:

$$\mu = \frac{\frac{4}{\nu^2}\sum_{n=1}^{\infty} C(\nu, n)_{2n} S_e(f_0)}{\sum_{n=1}^{\infty} C(\nu, n)_{2n} S_e(0)} = \frac{4}{\lambda\nu^2}\times$$

$$\times\sum_{n=1}^{\infty} \frac{\left(\frac{\nu}{2}\right)^2\left(1-\frac{\nu}{2}\right)^2\cdots\left(n-1-\frac{\nu}{2}\right)^2}{(n!)^2}\frac{{}_{2n}S_e(f_0)}{{}_{2}S_e(0)}. \qquad (4.45)$$

For an energy spectrum such as that of the single oscillatory circuit we get

$$\mu = \frac{4}{\lambda\nu^2}\sum_{n=1}^{\infty} \frac{\left(\frac{\nu}{2}\right)^2\left(1-\frac{\nu}{2}\right)^2\cdots\left(n-1-\frac{\nu}{2}\right)^2}{n!\,(n-1)!}\times$$

$$\times\frac{1}{n^2+x_0^2}, \qquad (4.46)$$

where $x_0 = f_0/\Delta f$ is the normalized intermediate frequency.

For convenience in analyzing let us look at the following auxiliary series

$$\mu_{np} = \frac{4}{\lambda\nu^2}\times$$

$$\times\sum_{n=1}^{\infty} \frac{\left(\frac{\nu}{2}\right)^2\left(1-\frac{\nu}{2}\right)^2\cdots\left(n-1-\frac{\nu}{2}\right)^2}{n!\,(n-1)!}\frac{1}{x_0^2}. \qquad (4.47)$$

Quantities n^2, x_0^2 are always positive, unless the terms of the positive theories are always greater than corresponding terms of series (4.46) and, consequently, $\mu < \mu_{np}$. The expression for the sum of the auxiliary series in closed form is found by the same method that is used to derive relationship (4.36). As a result we get

$$\mu_{\text{пр}} = \frac{4\Gamma(\nu)}{\lambda \nu^{2} \cdot \Gamma^{2}\left(\frac{\nu}{2}\right) x_0^2} = \frac{k_{\text{пр}}}{\lambda x_0^2}.$$

Note that:

1) the closer the sum of series (4.46) to the sum of auxiliary series (4.47), the greater will be x_0;

2) series (4.46) converges more rapidly than auxiliary series (4.47). Consequently, to obtain the sum of (4.46) with the required accuracy, we must consider a smaller or at least the same quantity of terms as we did in calculating auxiliary series (4.47). Calculations indicate that in order to obtain the sum of the auxiliary series with an accuracy to within no less than 5% it is sufficient to add the first four terms.

Figure 4.3 shows the values of loss coefficient μ calculated from formula (4.32) taking into account only the first four terms of the series for the three studied energy spectra. From the figure it follows that coefficient μ for an energy spectrum such as the single oscillatory circuit with $x_0 \geq 4$ is very close to the value determined by relationship (4.47) (shown as a dashed line). For the two other energy spectra relationships in closed form between loss coefficient μ and relative frequency x_0 were not obtained. For calculation at higher values of x_0 we must consider a greater number of terms in the series. In the case of $\nu = 2$, 4, ... expression (4.46) represents a finite number of terms and can be easily calculated.

Analysis of the curves in Fig. 4.3 shows us that:

1) loss coefficient μ decreases as the intermediate frequency increases. The rate of decrease is determined by the energy spectrum of the signal (reference voltage);

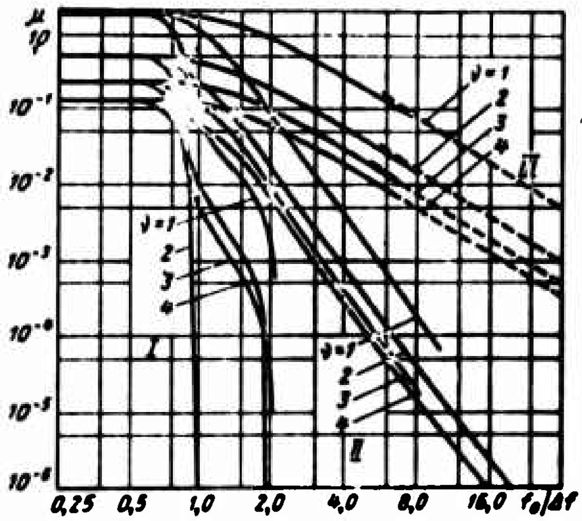

Fig. 4.3. Loss coefficient μ as a function of value of difference frequency and characteristic of nonlinear unit: I - rectangular signal spectrum; II - spectrum in the form of frequency of band-pass filter characteristic; III - spectrum in the form of frequency characteristic of oscillatory circuit.

2) loss coefficient μ depends on the form of the characteristic of the nonlinear unit, i.e., on parameter ν. This dependence becomes especially noticeable for signals whose energy spectrum is close to the rectangular.

2. Some Experimental Results

Some of the results obtained from experimental verification of theoretical proposals are interesting. In studying a functional correlator the parameters of the individual units were selected to be as close as possible to the models of signals described in §§4.1 and 2.2. With this goal two narrow-band processes with frequency-shifted energy spectra (signal and reference voltage) were shaped from the original gaussian noise using a method of filtration and frequency transformation. The voltage of the signal (through the controlled delay line) and the reference voltage were conducted to the input of a correlator with a multiplier or after the summation process were conducted to the input of the nonlinear element of a functional correlator. The nonlinear element was a series-connected point-contact semiconductor diode and a resistance load. The type of diode, load resistance, and insignificant forward diode bias were selected with the intention of obtaining an inertialess nonlinear characteristic when $\nu = 1$.

The measured output signal/noise ratio of the functional correlator in the absence of signal delay relative to the reference voltage was 4.9 dB higher than for the ideal correlator with analogous parameters (oscillatory power, averaging time, etc.). This confirms the above deduction that in the absence of extraneous noise the output signal/noise ratio of a functional correlator with a nonlinear element when $\nu = 1$ can be 5.8 dB better than that of an ideal correlator.

From relationship (4.16) it follows that the amplitude of the harmonic component of the output voltage of a functional correlator when $\nu = 1$ is equal to

$$A(\tau) = \frac{a\sigma_c}{\sqrt{2\pi}} r_n(\tau_d)$$

and does not depend on the power of the reference voltage. This property is useful in correlation measurements, since the power change in the reference voltage, which develops when the delay line is switched (due to an unsteady transmission coefficient) does not affect (or only slightly affects) the reference accuracy of correlation function values. Table 4.4 gives the experimental result for the studied unit.

Table 4.4.

σ, [В]	2	1	0,6	0,4	0,2	0,1
A [дБ]	0	0	0	−0.2	−1.1	−2.8

It is interesting to study the behavior of self-noise in correlators as a function of signal delay relative to the reference voltage. Theoretically this dependence is obtained from a rather ponderous expression (4.23). Figure 4.4 shows the results of experimentally measuring the relative spectral intensity of self-noise in a functional correlator for the intermediate frequency range as a function of delay.

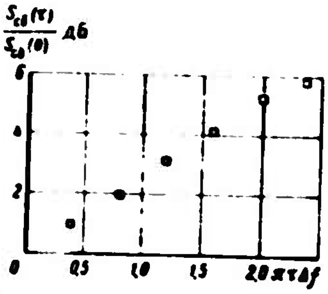

Fig. 4.4. Relative spectral intensity of self-noise in a correlator with a multiplier does not depend on signal delay. [дБ = dB].

These results confirm the advantage of using functional correlators of the given type in radio electronic systems. They can be used in developing specific devices.

FIFTH CHAPTER

NOISE CHARACTERISTICS OF FUNCTIONAL CORRELATORS BASED ON NONLINEAR ELEMENTS WITH AN EXPONENTIAL CHARACTERISTIC

5.1. REGULAR VOLTAGE COMPONENT AT CORRELATOR OUTPUT

Let us evaluate the effectiveness of a functional difference frequency correlator in which a nonlinear device with an exponential characteristic is used in place of the multiplier. The original premises and proposals concerning the nature of the signals, assumptions, symbols, and method of analysis are all analogous to those formulated in §§ 2.2, 4.1, and 4.2.

This problem should be studied, since elements and devices which have an exponential characteristic are widely used and since the properties of a functional correlator in this case are different from the properties of correlators of the ν-th power.

A nonlinear device can simply be regarded as a diode whose volt-ampere characteristic within the limits of a certain input voltage range can be described with sufficient accuracy by the dependence [29] $I = I_0(e^{vu} - 1)$. Let us examine the noise characteristics of a functional correlator with a nonlinear element in which the output voltage is proportional to the current which flows through the semiconductor diode, i.e.,

$$z = a(e^{\nu y} - 1). \tag{5.1}$$

The regular output voltage component is determined by averaging the response of the nonlinear element for the set

$$\overline{z(t)} = a(e^{\overline{\nu y(t)}} - 1). \tag{5.2}$$

The statistical average of $e^{\overline{\nu y}}$ is found by means of the expression for the unidimensional characteristic function of the random process y(t), which has a normal distribution [24]

$$e^{\overline{\nu y(t)}} = \exp\left[\frac{1}{2}\nu^2\sigma^2_{y(t)}\right].$$

If we consider (4.9) and (4.10) and use the Jacobi-Enger expansion [14, p. 333]

$$\exp(z\cos\theta) = \sum_{m=0}^{\infty} \varepsilon_m I_m(z)\cdot\cos m\theta,$$

where

$$\varepsilon_m = \begin{cases} 1 \text{ при } m = 0, \\ 2 \text{ при } m = 1, 2, 3, \ldots \end{cases}$$

[при = when]

$I_m(z)$ is the modified Bessel function, we get

$$e^{\overline{\nu y(t)}} = e^{\frac{\gamma^2}{2}\left(1 + \frac{\sigma_c^2}{\sigma_r^2} + \frac{\sigma_m^2}{\sigma_r^2}\right)} \times$$

$$\times \sum_{m=0}^{\infty} \varepsilon_m I_m\left[\gamma^2 \frac{\sigma_c}{\sigma_r} r_n(\tau_3)\right] \cos m\left[\omega_0 t - \omega_c\tau_3 - \zeta_n(\tau_3)\right]. \tag{5.3}$$

Here we have $\gamma^2 = \nu^2\sigma_r^2$.

For small values of the argument the following asymptotic representation of modified Bessel functions is valid [42, p. 248]:

$$I_\nu(x) \approx \frac{1}{\Gamma(\nu+1)}\left(\frac{x}{2}\right)^\nu \text{ при } 0 < x \ll 1.$$

[при = when]

Then, when $\gamma^2\sigma_{\text{с}} r_{\text{в}}(\tau_{\text{з}})\frac{1}{\sigma_{\text{г}}} \ll 1$ the regular component of the response of the nonlinear element, taking into account terms with an order of smallness no less than $\sigma_{\text{с}}/\sigma_{\text{г}}$ will have the form of

$$\overline{z(t)} = a\left\{e^{\gamma^2/2} - 1 + \gamma^2 e^{\gamma^2/2}\frac{\sigma_{\text{с}}}{\sigma_{\text{г}}} r_{\text{в}}(\tau_{\text{з}}) \times \right.$$
$$\left. \times \cos\left[\omega_0 t - \omega_{\text{с}}\tau_{\text{з}} - \zeta_{\text{в}}(\tau_{\text{з}})\right]\right\}.$$

Thus, when a weak signal and reference voltage have a combined effect on a nonlinear element with an exponential characteristic, a regular voltage component of intermediate frequency is obtained at the output, whose amplitude is proportional to the envelope of the cross-correlation function of the signal and reference voltage. This result is analogous to that obtained in the preceding chapter. Let us find this regular voltage component at frequency ω_0 by using relationships (5.2) and (5.3) and by assuming arbitrary signal amplitude

$$\left\{\overline{z(t)}\right\}_{\omega_0} = 2ae^{\frac{\gamma^2}{2}\left(1 + \frac{\sigma_{\text{с}}^2}{\sigma_{\text{г}}^2} + \frac{\sigma_{\text{ш}}^2}{\sigma_{\text{г}}^2}\right)} \times$$
$$\times I_1\left[\gamma^2\frac{\sigma_{\text{с}}}{\sigma_{\text{г}}} r_{\text{в}}(\tau_{\text{з}})\right]\cos\left[\omega_0 t - \omega_{\text{с}}\tau_{\text{з}} - \zeta_{\text{в}}(\tau_{\text{з}})\right].$$

If the correlation coefficient is measured from the amplitude of the regular component of frequency ω_0, then relative error in measuring the correlation function determined by formula (4.17) is expressed as

$$\Delta = \frac{I_1\left[\gamma^2 r_0(\tau_з)\frac{\sigma_c}{\sigma_r}\right]}{r_0(\tau_з)\, I_1\left[\gamma^2 \frac{\sigma_c}{\sigma_r}\right]} - 1.$$

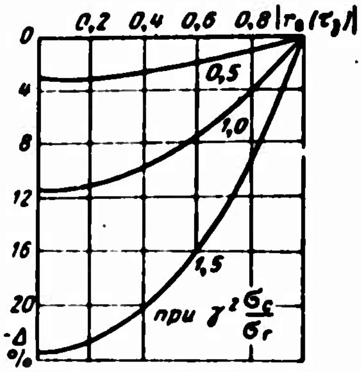

Fig. 5.1. Nonlinear error of measuring correlation function for nonlinear device with exponential characteristic.

Figure 5.1 shows measurement error found using this formula. This error diminishes rapidly as the power of the signal decreases, and does not exceed 3% when $\gamma^2\sigma_c/\sigma_r \leq 0.5$.

5.2. Signal/Noise Ratio at Correlator Output

Let us find noise intensity at the correlator output. For this purpose we will calculate the correlation function of voltage at the output of the nonlinear element from our knowledge of the response to the input effect. If we consider the presence of the nonstationary connection between the signal and reference voltage, then the function must be time-averaged. We get the following expression for the time-averaged value of the correlation function:

$$R_z(\tau) = a^2\left(\langle e^{\overline{\gamma y(t)}} e^{\overline{\gamma y(t+\tau)}}\rangle - \langle e^{\overline{\gamma y(t)}}\rangle - \langle e^{\overline{\gamma y(t+\tau)}}\rangle + 1\right). \qquad (5.4)$$

If we time average the expression (5.3) and consider our assumption of smallness for the signal and noise, we get

$$\langle e^{\overline{\gamma y(t)}}\rangle = e^{\frac{\gamma^2}{2}\left(1+\frac{\sigma_c^2}{\sigma_r^2}+\frac{\sigma_ш^2}{\sigma_r^2}\right)} \times I_0\left[\gamma^2\frac{\sigma_c}{\sigma_r} r_0(\tau_з)\right] \approx e^{\frac{\gamma^2}{2}}. \qquad (5.5)$$

Similarly we also get

$$<\overline{e^{\gamma y(t+\tau)}}> = e^{\gamma^2/2}. \tag{5.6}$$

If we use the two-dimensional characteristic function of the normal random process y(t) [24, p. 243] and consider (4.8), (4.9), and (4.10), then we find

$$\overline{e^{\gamma y(t)}\, e^{\gamma y(t+\tau)}} = \exp\left\{\frac{\gamma^2}{2}\left[\rho(t,0)+\rho(t+\tau,0)+2\rho(t,\tau)\right]\right\}.$$

In averaging this expression with respect to time we use the expansion for exponential time functions in series

$$e^x = \sum_{k=0}^{\infty}\frac{x^k}{k!}.$$

In averaging let us retain only those terms which have an order of smallness of no less than σ_c^2/σ_r^2 and $\sigma_ш^2/c_r^2$. As a result we get

$$\begin{aligned}
<\overline{e^{\gamma y(t)}\, e^{\gamma y(t+\tau)}}> &= e^{\gamma^2}\sum_{m=0}^{\infty}\varepsilon_m I_m\left[\gamma^2 r_r(\tau)\right]\times\\
&\times\cos m\omega_r\tau\cdot\left[1+\gamma^2\frac{\sigma_c^2}{\sigma_r^2}r_c(\tau)\cos\omega_c\tau+\gamma^2\frac{\sigma_ш^2}{\sigma_r^2}r_ш(\tau)\times\right.\\
&\left.\times\cos\omega_ш\tau\right]\left\{1+\frac{1}{2}\gamma^4\frac{\sigma_c^2}{\sigma_r^2}r_n^2(\tau_з)\,\cancel{\cos}\,\omega_c\tau+\right.\\
&+\frac{1}{2}\gamma^4\frac{\sigma_c^2}{\sigma_r^2}r_n(\tau+\tau_з)\,r_n(\tau_з-\tau)\cos\left[(\omega_c+\omega_r)\tau-\right.\\
&\left.-\zeta_n(\tau_з-\tau)-\zeta_n(\tau+\tau_з)\right]+\frac{1}{2}\gamma^4 r_n(\tau_з)\,r_n(\tau+\\
&+\tau_з)\frac{\sigma_c^2}{\sigma_r^2}\cos\left[\omega_c\tau+\zeta_n(\tau+\tau_з)-\zeta_n(\tau_з)\right]+\\
&+\frac{1}{2}\gamma^4\frac{\sigma_c^2}{\sigma_r^2}r_n(\tau_з)\,r_n(\tau_з-\tau)\cos\left[\omega_r\tau+\zeta_n(\tau_з)-\zeta_n(\tau_з-\tau)\right]+\\
&+\frac{1}{2}\gamma^4\frac{\sigma_c^2}{\sigma_r^2}r_n(\tau_з)\,r_n(\tau+\tau_з)\cos\left[\omega_r\tau+\zeta_n(\tau+\tau_з)-\zeta_n(\tau_з)\right]+\\
&\left.+\frac{1}{2}\gamma^4\frac{\sigma_c^2}{\sigma_r^2}r_n(\tau_з)\,r_n(\tau_з-\tau)\cos\left[\omega_c\tau+\zeta_n(\tau_з)-\zeta_n(\tau_з-\tau)\right]\right\};
\end{aligned}$$

Assuming that $\omega_0 \ll \omega_r \;\; \omega_c$, we separate the individual components corresponding to the low and intermediate frequency ranges. Then the time-averaged autocorrelation function of the response of the nonlinear element (5.4) with (5.5) and (5.6) considered will have the form of

$$R_s^{\text{н}\text{ч}}(\tau) = R_{0\text{д}} + R_{0\text{ш}} + R_{0\text{с}},$$

where we have:

$$R_{0\text{д}} = a^2(1 - 2e^{\gamma^2/2}) + a^2 e^{\gamma^2} I_0[\gamma^2 r_r(\tau)]; \tag{5.7}$$

$$\dot{R}_{0\text{ш}} = a^2 e^{\gamma^2} \gamma^2 \frac{\sigma_{\text{ш}}^2}{\sigma_r^2} I_1[\gamma^2 r_r(\tau)] r_{\text{ш}}(\tau) \times \\ \times \cos(\omega_r - \omega_{\text{ш}})\tau; \tag{5.8}$$

$$R_{0\text{с}} = a^2 e^{\gamma^2} \gamma^2 \frac{\sigma_c^2}{2\sigma_r^2} \{\gamma^2 r_{\text{в}}^2(\tau_3) I_0[\gamma^2 r_r(\tau)] \cos\omega_0\tau + \\ + 2I_1[\gamma^2 r_r(\tau)] r_c(\tau)\cos\omega_0\tau + \gamma^2 I_2[\gamma^2 r_r(\tau)] r_{\text{в}}(\tau_3 - \tau) \times \\ \times r_{\text{в}}(\tau_3 - \tau)\cos[\omega_0\tau + \zeta_{\text{в}}(\tau_3 - \tau) - \zeta_{\text{в}}(\tau_3 + \tau)] + \\ + \gamma^2 I_1[\gamma^2 r_r(\tau)] r_{\text{в}}(\tau_3) r_{\text{в}}(\tau + \tau_3) \cos[\omega_0\tau + \zeta_{\text{в}}(\tau_3) + \\ + \zeta_{\text{в}}(\tau + \tau_3)] + \gamma^2 I_1[\gamma^2 r_r(\tau)] r_{\text{в}}(\tau_3) r_{\text{в}}(\tau_3 - \tau) \times \\ \times \cos[\omega_0\tau + \zeta_{\text{в}}(\tau_3 - \tau) - \zeta_{\text{в}}(\tau_3)]\}. \tag{5.9}$$

In estimating the effectiveness of the functional correlator let us limit ourselves to the case where

$$\tau_3 = 0,\; \zeta_{\text{в}}(\tau) \equiv 0,\; \omega_{\text{ш}} = \omega_c,\; r_r(\tau) = r_c(\tau) = r_{\text{в}}(\tau) = r(\tau). \tag{5.10}$$

Then, if we write the moc fied Bessel functions in the form of series [42, p. 27]

$$I_\nu(z) = \left(\frac{z}{2}\right)^\nu \sum_{k=0}^{\infty} \frac{(z/2)^{2k}}{k!\,\Gamma(\nu + k + 1)}$$

and use recurrent relationships [42, p. 248], then for the energy spectra corresponding to the correlation functions of (5.7), (5.8) and (5.9) we get the following expressions:

$$S_{os} = a^2(1 - e^{\gamma^2/2})\delta(f) + a^2 e^{\gamma^2} \sum_{n=1}^{\infty} \frac{\left(\frac{\gamma^2}{2}\right)^{2n}}{(n!)^2}\,{}_{2n}S_c(f); \qquad (5.11)$$

$$S_{ow} = a^2\gamma^4 e^{\gamma^2} \frac{\sigma_w^2}{4\sigma_r^2} \sum_{n=1}^{\infty} \frac{\left(\frac{\gamma^2}{2}\right)^{2n-2}}{n!\,(n-1)!} [{}_{2n}S_{rw}(f - f_0) + {}_{2n}S_{rw}(f + f_0)]; \qquad (5.12)$$

$$S_{os} = a^2\gamma^4 e^{\gamma^2} \frac{\sigma_c^2}{4\sigma_r^2} \Big\{\delta(f - f_0) + \delta(f + f_0) + \sum_{n=1}^{\infty} \frac{\left(\frac{\gamma^2}{2}\right)^{2n-2}}{[(n-1)!]^2} \left(1 + \frac{\gamma^2}{2n}\right)^2 [{}_{2n}S_c(f - f_0) + {}_{2n}S_c(f + f_0)]\Big\}, \qquad (5.13)$$

where $\delta(f)$ is the delta-function of Dirac;

$${}_{2n}S_c(f) = \int_{-\infty}^{\infty} r^{2n}(\tau)\, e^{-j2\pi f\tau}\, d\tau;$$

$${}_{2n}S_{rw}(f) = \int_{-\infty}^{\infty} r^{2n-1}(\tau)\, r_w(\tau)\, e^{-j2\pi f\tau}\, d\tau.$$

On the basis of these expressions the signal/noise ratio at the output of a narrow-band averaging filter engaged after the nonlinear element, will be

$$\left(\frac{P_s}{P_w}\right)_{os} = \frac{2}{B_y}\Big\{\frac{\sigma_r^2}{\sigma_c^2} \sum_{n=1}^{\infty} \frac{\left(\frac{\gamma^2}{2}\right)^{2n-2}}{(n!)^2} \times {}_{2n}S_c(f_0) + \sum_{n=1}^{\infty} \frac{\left(\frac{\gamma^2}{2}\right)^{2n-2}}{[(n-1)!]^2} \left(1 + \frac{\gamma^2}{2n}\right)^2 \times [{}_{2n}S_c(0) + {}_{2n}S_c(2f_0)] + \frac{\sigma_w^2}{\sigma_c^2} \times \sum_{n=1}^{\infty} \frac{\left(\frac{\gamma^2}{2}\right)^{2n-2}}{(n-1)!\,n!} [{}_{2n}S_{rw}(0) + {}_{2n}S_{rw}(2f_0)]\Big\}^{-1}. \qquad (5.14)$$

where B_y is the effective pass band of the averaging filter of a functional correlator, determined by relationship (2.8).

For an ideal difference frequency correlator based on pure signal multiplication the output signal/noise ratio is determined by relationship (2.7). Let us give expression (5.14) a form which is more convenient for comparison

$$\left(\frac{P_c}{P_ш}\right)_{вх} = \frac{2}{B_y}\left\{\lambda\left(1+\mu\frac{\sigma_r^2}{\sigma_c^2}\right)[{}_1S_c(0)+ \right.$$

$$\left. + {}_1S_c(2f_0)] + \beta\frac{\sigma_ш^2}{\sigma_c^2}[{}_1S_{rш}(0) + {}_1S_{rш}(2f_0)]\right\}^{-1}. \qquad (5.15)$$

where we have:

$$\mu = \frac{\sum\limits_{n=1}^{\infty}\frac{\left(\frac{\gamma^2}{2}\right)^{2n-2}}{(n!)^2}\,{}_{2n}S_c(f_0)}{\sum\limits_{n=1}^{\infty}\frac{\left(\frac{\gamma^2}{2}\right)^{2n-2}}{[(n-1)!]^2}\left(1+\frac{\gamma^2}{2n}\right)^2[{}_{2n}S_c(0) + {}_{2n}S_c(2f_0)]}. \qquad (5.16)$$

$$\lambda = \sum_{n=1}^{\infty}\frac{\left(\frac{\gamma^2}{2}\right)^{2n-2}}{[(n-1)!]^2}\left(1+\frac{\gamma^2}{2n}\right)^2\frac{{}_{2n}S_c(0) + {}_{2n}S_c(2f_0)}{{}_1S_c(0) + {}_1S_c(2f_0)}, \qquad (5.17)$$

$$\beta = \sum_{n=1}^{\infty}\frac{\left(\frac{\gamma^2}{2}\right)^{2n-1}}{n!\,(n-1)!}\,\frac{{}_{2n}S_{rш}(0) + {}_{2n}S_{rш}(2f_0)}{{}_1S_{rш}(0) + {}_1S_{rш}(2f_0)}. \qquad (5.18)$$

The introduced loss coefficients for the functional correlator μ, λ, β have the same physical content as in the problem for the correlator of the ν-th power (see Chapter 4).

The relationships which we have obtained depend on the shape of the energy spectra of the signals and parameter $\gamma = \nu\sigma_r$, i.e., on the characteristic of the nonlinear element and the power of the reference voltage.

5.3. EFFECTIVENESS OF FUNCTIONAL CORRELATOR

In order to estimate the effectiveness of the correlator we examine loss coefficients. Loss coefficient β is analyzed for two cases.

1. The spectrum of extraneous noise is much wider than that of the signal. For this case, as shown in § 4.4,

$$ {}_{эп}S_{r\,ш}(f) \approx S^{n}_{ш}(f) $$

and the expression for coefficient β based on (5.18) will have the form

$$ \beta = \sum_{n=0}^{\infty} \frac{\left(\frac{\gamma^2}{2}\right)^{2n}}{n!\,(n+1)!} = \frac{2}{\gamma^2} I_1(\gamma^2). \qquad (5.19) $$

The dependence of loss coefficient β on parameter γ is shown in Fig. 5.2 (curve a).

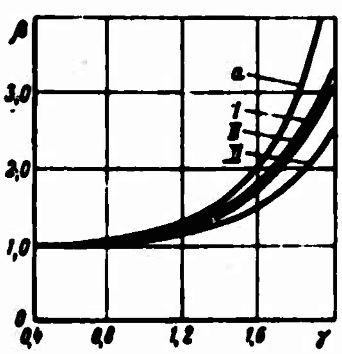

Fig. 5.2. Loss coefficient β as a function of the form of a nonlinear device characteristic: a - maximal value; I - spectrum in the form of frequency characteristic of band-pass filter; II - rectangular signal spectrum; III - spectrum in the form of frequency characteristic of oscillatory circuit.

2. The energy spectra of the extraneous noise and the signal are identical.

In a case where $r_ш(\tau) = r(\tau),\ {}_{эп}S_{ш}(f) = {}_{эп}S_c(f)$, from expression (5.18) we get

$$\beta = 1 + \sum_{n=1}^{\infty} \frac{\left(\frac{\gamma^2}{2}\right)^{2n}}{n!\,(n+1)!} \cdot \frac{{}_{2n+2}S_c(0) + {}_{2n+2}S_c(2f_0)}{{}_2S_c(0) + {}_2S_c(2f_0)}. \qquad (5.20)$$

In practical difference-frequency correlator systems intermediate frequency f_0 is selected to be much wider than the energy spectrum of the reference voltage. In this case

$${}_{2n+2}S_c(2f_0) \ll {}_{2n+2}S_c(0)$$

and loss coefficient β can be represented in the form of

$$\beta = \sum_{n=0}^{\infty} \frac{\left(\frac{\gamma^2}{2}\right)^{2n}}{n!\,(n+1)!} \cdot \frac{{}_{2n+2}S_c(0)}{{}_2S_c(0)}. \qquad (5.21)$$

In the case of an energy spectrum such as that in the unidimensional oscillatory circuit we get:

$$\frac{{}_{2n+2}S_c(0)}{{}_2S_c(0)} = \frac{1}{n+1},$$

$$\beta = \sum_{n=0}^{\infty} \frac{\left(\frac{\gamma^2}{2}\right)^{2n}}{[(n+1)!]^2} = \left(\frac{2}{\gamma^2}\right)^2 [I_0(\gamma^2) - 1]. \qquad (5.22)$$

For a rectangular energy spectrum or an energy spectrum such as that of the band-pass filter the values of the loss coefficient are found by adding the first four terms of the series (5.21). This approximation is possible for a range of γ values from 0 to 2.0, where the series converges very rapidly. Since ${}_{2n+2}S_c(0) \leq {}_2S_c(0)$, then the maximal value of (5.21) is determined by a series with majorized terms

$$\beta_{ap} = \sum_{n=0}^{\infty} \frac{\left(\frac{\gamma^2}{2}\right)^{2n}}{n!\,(n+1)!} = \frac{2}{\gamma^2} I_1(\gamma^2).$$

which corresponds to the expressions of (5.19) for a loss coefficient in the case of wide-band extraneous noise. Figure 5.2 shows the graphic dependences of loss coefficient β on parameter γ. From this figure it follows that coefficient β beginning at a certain value of $\gamma \geq 1.2$ increases very rapidly, which leads to an abrupt decline in the output signal/noise ratio.

Let us examine loss coefficient λ. Assuming that the intermediate frequency is sufficiently high in comparison to the width of the reference voltage energy spectrum, then from (5.17) we get the following expression:

$$\lambda = \sum_{n=0}^{\infty} \frac{\left(\frac{\gamma^2}{2}\right)^{2n}}{(n!)^2} \left(1 + \frac{\gamma^2}{2n+2}\right)^2 \frac{{}_{2n+2}S_c(0)}{{}_{2}S_c(0)}. \qquad (5.23)$$

The dependence of λ on parameter γ for the three types of energy spectra plotted from formula (5.23) is shown in Fig. 5.3.

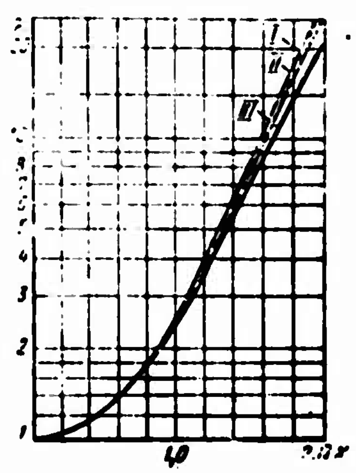

Fig. 5.3. Dependence of loss coefficient λ on the form of the characteristic of the nonlinear device: I - rectangular signal spectrum; II - spectrum in the form of frequency characteristic of band-pass filter; III - spectrum in the form of frequency characteristic of oscillatory circuit.

The maximal value of series (5.23) can be found by means of a series with majorized terms

$$\lambda_{\text{мр}} = \sum_{n=0}^{\infty} \frac{\left(\frac{\gamma^2}{2}\right)^{2n}}{(n!)^2} \left(1 + \frac{\gamma^2}{2n+2}\right)^2 = 2\left[I_1(\gamma^2) + I_0(\gamma^2)\right] - 1.$$

The values of λ obtained from this curve when $\gamma \geq 2.0$ are very high because of the great weight acquired by second-order terms.

Let us examine loss coefficient μ. From (5.16) and (5.17) we get the following expression for the case of great frequency intervals:

$$\mu = \frac{1}{\lambda} \sum_{n=1}^{\infty} \frac{\left(\frac{\gamma^2}{2}\right)^{2n-2}}{(n!)^2} \cdot \frac{{}_{2n}S_c(f_0)}{{}_2S_c(0)}.$$

For an energy spectrum such as that of the single oscillatory circuit we have

$$\frac{{}_{2n}S_c(x_0)}{{}_2S_c(0)} = \frac{n}{n^2 + x_0^2},$$

$$\mu = \frac{1}{\lambda} \sum_{n=1}^{\infty} \frac{\left(\frac{\gamma^2}{2}\right)^{2n-2}}{(n-1)!\, n!} \cdot \frac{1}{n^2 + x_0^2}, \qquad (5.24)$$

where $x_0 = f_0/\Delta f$ is normalized intermediate frequency. As intermediate frequency increases the sum of series (5.24) approaches the value of the series with majorized terms

$$\mu_{\text{пр}} = \frac{1}{\lambda} \sum_{n=1}^{\infty} \frac{\left(\frac{\gamma^2}{2}\right)^{2n-2}}{(n-1)!\, n!} \cdot \frac{1}{x_0^2} = \frac{\beta_{\text{пр}}}{\lambda x_0^2}. \qquad (5.25)$$

Figure 5.4 shows loss coefficient μ as a function of the normalized frequency and parameter γ for the three energy spectra of the signal. The calculations were made according to formula (5.16) with the four terms of the series considered. The value of loss coefficient μ for an energy spectrum such as that in the single oscillatory circuit at high normalized frequency values is shown as dashed lines. No expression was obtained for loss coefficient μ in a closed form for a rectangular energy spectrum or a spectrum such as that of the band-pass filter. For the

band-pass filter type of energy spectrum we can assume that the future behavior of the curves in Fig. 5.4 can also be determined by the asymptotic curves.

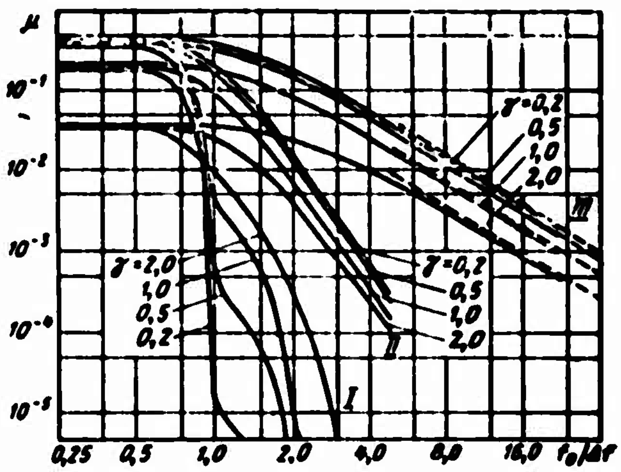

Fig. 5.4. Loss coefficient μ as a function of the difference frequency value and the characteristic of the nonlinear device: I - rectangular signal spectrum; II - spectrum in the form of difference characteristic of band-pass filter; III - spectrum in the form of difference characteristic of oscillatory circuit.

5.4. SELECTING PARAMETERS AND ELEMENTS IN SIMPLEST FUNCTIONAL CORRELATORS

The number of functional correlator parameters which can be varied as the system is being constructed might include selection of the nonlinear element depending on the form of its characteristic and, in a number of cases, selection of the difference frequency on which the regular component at the correlator output is separated. The correlator parameters should be selected to reduce as much as possible the effect of extraneous noise on work. The parameters can be selected on the basis of relationships (4.13)-(4.34) and (5.15)-(5.18).

When the spectral density of the signals has a fixed form, intermediate frequency f_0 must be selected such that losses in the

signal/noise output ratio from the detected noise of the reference voltage are small. From relationships (4.31) and (5.15) it is apparent that the magnitude of coefficient μ, which depends primarily on the intermediate frequency value, should satisfy the inequality

$$\mu < \frac{e_c^2}{e_p^2}.$$

If the signal is very low in power, then we must select a high intermediate frequency in order to fulfill this inequality, which is not always possible in a wide signal spectrum. In this case the use of balanced systems (see § 2.3) is of particular interest. In the output oscillation of an ideal balanced system there are no harmonic components on the reference voltage frequency or its harmonics. In an actual balanced system, due to the fact that the characteristics of the nonlinear element are not identical, the transformers are asymmetrical, etc., these harmonic components are present, although weakened by a factor of p (where p is the suppression coefficient in the balanced system). In balanced systems in which special control [tuning] elements are used the suppression coefficient can be brought to within 50-60 dB [63].

Thus, in a correlator which is balanced relative to the reference voltage the noise of the detected reference signal and, consequently, the coefficient of losses μ are reduced by p times. With respect to the input signal the parallel and sequential balance systems (see Fig. 2.7) represent "key" systems in the presence of a high reference voltage, and work in time with the reference voltage, which corresponds to the nonlinear element with a characteristic of $\nu = 1$.

Thus, in calculating the output signal/noise ratio of a balanced functional correlator we can assume the parameter of the nonlinear element to be $\nu = 1$ and the value of ν to be reduced p times.

In functional correlators there are semiconductor elements and devices whose characteristic can be approximated with a sufficient degree of accuracy by power and exponential functions. The approximation of the nonlinear characteristic of a semiconductor diode by an exponential dependence (5.1) is correct for a limited segment of the change in input voltage. Thus, the relationship obtained in Chapter 5 will be valid for an actual system with a semiconductor diode in a limited range of values for parameter γ. According to the data of monograph [29] silicon pulse semiconductor diodes have a region of the dependence of (5.1) in an input voltage below 0.6 V and have a coefficient of $\nu = 20-25$. If we assume that the effective value of the gaussian process is on the average 1/3 of that of the greatest peak (which is correct with a probability of 0.97), then we find that the relationships derived in Chapter 5 are valid for an actual system with a semiconductor diode when $\gamma = \nu\sigma_{\Gamma} \leq 4-5$.

Let us estimate possible losses in the signal/noise ratio for a segment in which this approximation is correct for the semiconductor diode characteristic, namely $\gamma = 3$. In actual systems with narrow-band averaging end filters the energy losses of the functional correlator can be determined, if the intermediate frequency is correctly selected, by the loss coefficient relative to extraneous noise. For "white" extraneous noise when $\gamma = 3$ value of the loss coefficient based on (5.19) will be β 230. This means that at a reference voltage power corresponding to this value of γ the energy losses of the output signal/noise ratio of a functional correlator will be 24 dB, which is not permissible.

As the reference voltage continues to increase the characteristic of the diode according to [29] is close to an exponential function with an exponent of 3/2 and, consequently, it is better to use an approximation in the form of a half-wave characteristic of the ν-th power. Hence it follows that in an actual system with a semiconductor diode the increase in the loss coefficient occurs

only up to a certain reference voltage power. When this threshold value is exceeded the coefficients either remain constant or decrease in value. For this reason it is best to deliver forward bias voltage to the semiconductor diode so that output will be on the desirable part of the nonlinear characteristic.

In using a superhigh-frequency diode as the nonlinear element we must keep in mind that the characteristic of this diode is close to square law. Consequently, if the intermediate frequency is sufficiently high we need not expect a great decline in the signal/noise ratio of the difference frequency functional correlator as compared to the ideal correlator.

SIXTH CHAPTER

EFFECT OF LINEAR CHANNELS ON CORRELATION PROCESSING OF NOISE SIGNALS

6.1. ANALYSIS OF LINEAR TRANSFORMATION IN CORRELATED SYSTEMS

Here and henceforth we attempt to study the effect of linear transformations on the characteristics of a correlated system. Generalized block diagrams of correlated systems of different types (Fig. 3.3, Fig. 3.8) contain a filter in a general channel Φ_0 and a filter in correlator channels Φ_1 and Φ_2. There is no Φ_0 filter in systems such as the passive radar and sonar systems. Signals and noise pass along two channels containing filters Φ_1 and Φ_2 to the correlator. Systems which store the emitted signal, for example, systems such as the correlated radar set, have filter Φ_0. In addition, the channel along which reference voltage is supplied to the correlator is free of noise.

After the presence of a signal has been detected the main problem becomes that of measuring delay time of the signal in one channel in relation to the other. Let us study the effect of linear filters Φ_1 and Φ_2 on detecting ability in measuring delay in a correlated system. We will assume that filter Φ_0 does not introduce distortion. The studied problem is solved by means of passive correlated systems (see § 3.2), for which our analysis is also made.

Figure 3.5 shows the generalized block diagram of a correlated shift meter during noise signals $s_1(t)$ and $s_2(t)$ against a background of extraneous noises $n_1(t)$ and $n_2(t)$, which are uncorrelated with each other or with the noise signals. The signals and the noise are added, passed through the selective linear devices Φ_1 and Φ_2 with transmission coefficients $K_1(if$ and $K_2(if)$ and enter the multiplier input. At the multiplier output averaging is done by a narrow-band filter. Let us designate $x_1(t)$ and $x_2(t)$ as the studied random processes $s_1(t)$ and $s_2(t)$ transformed by the filters; $y_1(t)$, $y_2(t)$ - interference $n_1(t)$ and $n_2(t)$ transformed by the filters.

At the multiplier output we get the process

$$z=[x_1(t)+y_1(t)][x_2(t)+y_2(t)]=x_1x_2+y_1y_2+x_1y_2+x_2y_1.$$

The first term is the product of correlated random processes x_1 and x_2, and can be written in the form of the sum of the regular and fluctuating components. The regular component determines the value of the correlation function of random processes x_1 and x_2. The fluctuating component represents noise at the output of the correlation system, which is caused by the random nature of the signals, i.e., self-noise. Usually noise of this type is ignored for two reasons:

1) the band of the averaging filter is much narrower than the energy spectra of signals at the output of the linear filters;

2) in processing weak signals the power of the self-noise at the output of the averaging filter is much smaller than that of extraneous noise and interference.

Considering the above statements, the correlation function of a random process at the multiplier output should have the form of

$$R_z(\tau)=(\overline{x_1x_2})^2+\overline{x_1x_{1\tau}}\,\overline{y_2y_{2\tau}}+\overline{x_2x_{2\tau}}\cdot\overline{y_1y_{1\tau}}+$$
$$+\overline{y_1y_{1\tau}}\,\overline{y_2y_{2\tau}}=R^2_{x_1x_2}+R_{x_1}R_{y_2}+R_{x_2}R_{y_1}+R_{y_1}R_{y_2}. \qquad (6.1)$$

Let us calculate the first term under the assumption that signals $s_1(t)$ and $s_2(t)$ differ from one another only by a time shift, i.e., $s_2(t) = s_1(t - \tau)$. Let us represent an input signal periodically extended in an interval [0, T] in the form of the Fourier series:

$$s_1(t)=\sum_{n=0}^{\infty}\varepsilon_n\cos(2\pi f_n t-\varphi_n), \qquad (6.2)$$

where

$$f_n=nf_0=\frac{n}{T},$$

while amplitudes ε_n and phases ϕ_n are random quantities.

It has been shown [35] that if we examine a group of occurrences then phase angles ϕ_n and amplitudes ε_n are independent quantities. Here

$$\overline{\frac{1}{2}\varepsilon_n^2}=G(f_n)\frac{1}{T},$$

where $G(f_n)$ is the spectral density of signal power calculated for positive frequencies.

If we consider (6.2), then the signals at the linear filter outputs can be represented in the form of:

$$x_p(t)=\sum_{n=0}^{\infty}\varepsilon_n K_p(f_n)\cos[2\pi f_n t-\varphi_n-\varphi_p(f_n)],$$

where p is an index equal to 1 or 2 for the channel of filter Φ_1 or Φ_2, respectively; $K_p(f)$, $\phi_p(f)$ are the amplitude-frequency

and phase-frequency characteristics of linear filters Φ_1 and Φ_2 (for corresponding subscripts).

The constant voltage component at the multiplier output depends on the delay and is determined by the value of the correlation function of processes $x_1(t)$ and $x_2(t)$:

$$R_{x_1x_2} = \overline{x_1x_2} = \sum_{n=0}^{\infty} G(f_n) K_1(f_n) K_2(f_n) \frac{1}{T} \cos[2\pi f_n\tau + \varphi_2(f_n) - \varphi_1(f_n)].$$

In directing the period for achieving T toward infinity, we come to the following integral relationship:

$$R_{x_1x_2} = \int_0^{\infty} G(f) K_1(f) K_2(f) \cos[2\pi f\tau + \varphi_2(f) - \varphi_1(f)]\,df.$$

Note that if the spectra of studied signals are shifted, then at the multiplier output we will have harmonic voltage and it will be necessary to engage the intermediate difference frequency filter.

It is evident that if the signal which is delayed for time τ is signal $s_1(t)$, then this can be considered in the above equality by reversing the sign of τ. Further calculations are reduced to integration on the plane of the complex variable, and this sign change proves to be essential in the selection of the integration contour. Henceforth we will assume everywhere that the delay is positive, and the expression for the correlation function will be represented in the form of

$$\left.\begin{matrix} R_{12}(\tau) \\ R_{21}(\tau) \end{matrix}\right\} = \int_0^{\infty} G(f) K_1(f) K_2(f) \cos\left[2\pi f\tau + \begin{matrix} \varphi_1(f) - \varphi_2(f) \\ \varphi_2(f) - \varphi_1(f) \end{matrix}\right] df. \quad (6.3)$$

where $R_{12}(\tau)$ is the constant component of the output voltage of the multiplier if signal $s_1(t)$ is delayed by time τ relative to $s_2(t)$; $R_{21}(\tau)$ is the same, although signal $s_2(t)$ is delayed by time τ relative to $s_1(t)$.

If the energy spectrum of the input signal is much wider than the pass band of the linear filters, i.e., if in the filter band $G(f) = G_0$, then the normalized cross-correlation function of random processes $x_1(t)$ and $x_2(t)$ based on (6.3) can be written in the form of

$$\left.\begin{matrix} r_{12}(\tau) \\ r_{21}(\tau) \end{matrix}\right\} = \frac{1}{\sigma_1\sigma_2}\int_0^\infty K_1(f)K_2(f)\cos\left[2\pi f\tau + \begin{matrix}\varphi_1(f)-\varphi_2(f) \\ \varphi_2(f)-\varphi_1(f)\end{matrix}\right]df. \qquad (6.4)$$

where σ_1 and σ_2 are determined by the relationship $\sigma_p = \sqrt{\int_0^\infty K_p^2(f)\,df}$ when the index is equal to 1 or 2.

It is obvious that when $K_1(if) = K_2(if)$ the relationships of (6.4) give us the expression for an autocorrelation function of a linear filter excited by white noise.

Let us assume that random processes s_1, s_2, n_1, n_2 are "white" noises with spectral power densities $X_{s1} = S_{s2} = S_c$, $S_{n1} = S_{n2} = S_ш$. Signals $s_1(t)$ and $s_2(t)$ differ by a time shift of $\tau_з$. Then, on the basis of (6.3) and (6.4) the separate terms of (6.1) will be equal to

$$R_{x_1x_2} = S_c\sigma_1\sigma_2 r_{12}(\tau_з),$$

$$R_{x_1} = S_c\sigma_1^2 r_1(\tau),\ R_{x_2} = S_c\sigma_2^2 r_2(\tau),\ R_{y_1} = S_ш\sigma_1^2 r_1(\tau),$$

$$R_{y_2} = S_ш\sigma_2^2 r_2(\tau),$$

where σ_1 and σ_2 are the dispersions of the noise processes at the outputs of filters Φ_1 and Φ_2 excited by "white" noise of a single spectral intensity; $r_1(\tau)$, $r_2(\tau)$ are the normalized autocorrelation functions of the noise processes at the outputs of filters Φ_1 and Φ_2,

respectively, when these filters are excited by "white" noise; $r_{в3}(\tau_3)$ is the value of the normalized cross-correlation function of processes $x_1(t)$ and $x_2(t)$, which is determined by formula (6.4) for delay $\tau = \tau_3$.

Considering this assumption, relationship (6.1) will have the form of

$$R_z = \sigma_1^2 \sigma_2^2 [S_c^2 r_{в3}^2(\tau_3) + (S_ш^2 + 2S_c S_ш) r_1(\tau) r_2(\tau)].$$

and the corresponding energy spectrum

$$S_z(f) = \sigma_1^2 \sigma_2^2 [S_c^2 r_{в3}^2(\tau_3) \delta(f) + (S_ш^2 + 2S_c S_ш) \int_{-\infty}^{\infty} r_1(\tau) r_2(\tau) \times$$
$$\times e^{-j2\pi f\tau} d\tau].$$

Obviously the spectaal power density of the noise, which is determined by the second term of the expression in brackets, will have an almost constant quantity within the limits of the pass band of the narrow-band averaging filter, which has a maximal transmission coefficient in the zero (or difference) frequency range. Then the signal/noise ratio at the output of the averaging filter will be equal to

$$\frac{P_c}{P_ш} = \frac{S_c^2}{S_ш^2} \frac{r_{в3}^2(\tau_3)}{\left(1 + 2\frac{S_c}{S_ш}\right) B_y \int_{-\infty}^{\infty} r_1(\tau) r_2(\tau) d\tau}, \qquad (6.5)$$

where $B_y = \frac{\int_{-\infty}^{\infty} |H_y(f)|^2 df}{H_{max}^2}$ is the effective band of the averaging filter.

The obtained relationships are then used to analyze the properties of correlation systems with linear filters of different types.

6.2. CROSS-CORRELATION FUNCTION OF PROCESSES AT OUTPUTS OF n-STAGE LINEAR FILTERS

Let us calculate the normalized cross-correlation function of voltages at the output of linear devices whose transmission coefficients are analogous to n-stage resonance amplifiers with a single circuit, adjusted to the same frequency for all stages, i.e.,

$$K_p(if) = \left[1 + i\frac{2(f-f_p)}{B_p}\right]^{-n},$$

where p is an index equal to 1 or 2; f_1 and f_2 are the central frequencies of the filters; B_1 and B_2 are the pass bands of each stage at a level of -3 dB.

It is very tempting to use the mathematical integration apparatus on the plane of the complex variable in calculating the cross-correlation function of (6.4). Here, however, we must consider certain fine points associated with approximate expressions used in radio engineering for the transmission coefficients of linear filters. The complex transmission coefficient for a linear circuit should satisfy the relationship

$$K(-if) = K^*(if).$$

The approximate expressions above for transmission coefficients of n-cascade linear amplifiers do not satisfy these requirements for a complex transmission coefficient. In order to solve the problem of using an approximate expression for the transmission coefficient which would be valid for the positive frequency range, let us represent relationship (6.4) in another form. Note that in the case of narrow-band linear filters Φ_1 and Φ_2 the lower limits in the integrals of (6.4) can be assumed infinite. Then

$$\left.\begin{matrix} r_{12}(\tau) \\ r_{21}(\tau) \end{matrix}\right\} = \frac{1}{\sigma_1\sigma_2} \mathrm{Re} \int_{-\infty}^{\infty} K_1(if) K^*_2(if) \begin{matrix} e^{-i2\pi f\tau} \\ e^{i2\pi f\tau} \end{matrix} df. \qquad (6.6)$$

where σ_1, σ_2 are equal to $\sigma_p = \sqrt{\int_{-\infty}^{\infty} |K_p(if)|^2 df}$ when p = 1; 2.

For the sake of abbreviation we write

$$\left.\begin{matrix} I_1 \\ I_2 \end{matrix}\right\} = \mathrm{Re} \int_{-\infty}^{\infty} K_1(if) K^*_2(if) \begin{matrix} e^{-i2\pi f\tau} \\ e^{i2\pi f\tau} \end{matrix} df.$$

We will use the first of these integrals to demonstrate our calculation method. After substituting the value of the transmission coefficients, we get

$$I_1 = \mathrm{Re} \left(\frac{B_1 B_2}{2}\right)^n \times$$

$$\times \int_{-\infty}^{\infty} \frac{e^{-i2\pi f\tau}}{\left[f - \left(f_1 + i\frac{B_1}{2}\right)\right]^n \left[f - \left(f_2 - i\frac{B_2}{2}\right)\right]^n}.$$

In integrating on the complex frequency plane we form a contour consisting of the real axis and a semicircle with an infinite radius in the lower half-plane. Then, according to the Jordan lemma the integral for an arc with an infinite radius is equal to zero, while the unknown integral according to the Cauchy theorem is found from the sum of remainders for the bands in the lower half-plane:

$$I_1 = \mathrm{Re} \left(\frac{B_1 B_2}{2}\right)^n \frac{(-2\pi i)}{(n-1)!} \times$$

$$\times \frac{d^{n-1}}{df^{n-1}} \left\{ \frac{e^{-i2\pi f\tau}}{\left[f - \left(f_1 + i\frac{B_1}{2}\right)\right]^n} \right\} \Bigg|_{f = f_2 - \frac{iB_2}{2}}.$$

If we use the Leibniz formula to calculate the derivative of the product, we get

$$\begin{aligned} I_1 = {} & \left(\frac{\pi B_1 B_2}{B_1+B_2}\right)\frac{e^{-\pi\tau B_2}}{(n-1)!}\left(\frac{\pi\tau B_1 B_2}{B_1+B_2}\right)^{n-1}\times \\ & \times\sum_{l=0}^{n-1}\frac{(n+l-1)!}{l!\,(n-l-1)!}\left[\pi\tau(B_1+B_2)\right]^{-l}\times \\ & \times\left\{1+\left[\frac{2(f_2-f_1)}{B_1+B_2}\right]^2\right\}^{-(n+l)/2}\times \\ & \times\cos\left[2\pi f_2\tau+(n+l)\operatorname{arctg}\frac{2(f_2-f_1)}{B_1+B_2}\right]. \end{aligned} \tag{6.7}$$

According to (6.6) the power of the noise process at the output of filter Φ_1 is found from (6.7) when $f_1 = f_2$, $B_1 - B_2$, $\tau = 0$. Then we get

$$\sigma_1^2 = \pi B_1 2^{-2n+1}\frac{(2n-2)!}{[(n-1)!]^2}. \tag{6.8}$$

Integral I_2 is calculated analogously, except that the integration contour consists of the actual axis of the frequencies and the semicircle of infinite radius in the upper half-plane. The calculations give us

$$\begin{aligned} I_2 = {} & \left(\frac{\pi B_1 B_2}{B_1+B_2}\right)\frac{e^{-\pi\tau B_1}}{(n-1)!}\left(\frac{\pi\tau B_1 B_2}{B_1+B_2}\right)^{n-1}\times \\ & \times\sum_{l=0}^{n-1}\frac{(n+l-1)!}{l!\,(n-l-1)!}\left[\pi\tau(B_1+B_2)\right]^{-l}\times \\ & \times\left\{1+\left[\frac{2(f_2-f_1)}{B_1+B_2}\right]^2\right\}^{-(n+l)/2}\times \\ & \times\cos\left[2\pi f_1\tau-(n+l)\operatorname{arctg}\frac{2(f_2-f_1)}{B_1+B_2}\right], \end{aligned} \tag{6.9}$$

$$\sigma_2^2 = \pi B_2 2^{-2n+1}\frac{(2n-2)!}{[(n-1)!]^2}. \tag{6.10}$$

If we substitute the found quantities in (6.6) and designate

$$c=\frac{B_1+B_2}{2};\; b=\frac{2(f_2-f_1)}{B_1+B_2};\; a=\frac{B_2}{B_1}, \tag{6.11}$$

then for the normalized cross-correlation function of random processes at the outputs of n-stage linear amplifiers we get relationships:

$$\left.\begin{aligned} r_{12}(\tau) &= e^{-2\pi a\tau c/(1+a)} \\ r_{21}(\tau) &= e^{-2\pi\tau c/(1+a)} \end{aligned}\right\} \frac{(n-1)!\, a^{n-0.5}}{2^{-2n+1}(2n-2)!\,(1+a)^{2n-1}} \times$$

$$\times \sum_{l=0}^{n-1} \frac{l_+!}{l!\, l_-!} (2\pi\tau c)^{l_-} (1+b^2)^{-(n+l)/2} \times$$

$$\times \left\{ \begin{aligned} &\cos[2\pi f_2\tau + (n+l)\,\mathrm{arctg}\, b] \\ &\cos[2\pi f_1\tau - (n+l)\,\mathrm{arctg}\, b] \end{aligned} \right\}, \tag{6.12}$$

where $l_+ = (n+l-1)$; $l_- = (n-l-1)$.

From these relationships when $f_1 = f_2 = f_0$, $a = 1$, $b = 0$, $B_1 = B_2 = B = c$ we get the expression for the normalized auto-correlation function of the noise process at the output of an n-stage resonance amplifier excited by "white" noise:

$$r(\tau) = \frac{(n-1)!}{(2n-2)!} \sum_{l=0}^{n-1} \frac{l_+!}{l!\, l_-!} (2\pi|\tau| B)^{l_-} e^{-\pi|\tau| B} \cos 2\pi f_0\tau. \tag{6.13}$$

Now let us examine correlation systems whose linear blocks have nonidentical characteristics.

6.3. EFFECT OF NONIDENTICAL CHARACTERISTICS OF LINEAR DEVICES ON THE SHAPE OF THE CROSS-CORRELATION FUNCTION

The noise processes at the outputs of linear amplifiers can be considered narrow-band. Thus it is logical to write the relationships of (6.12) so that they will be represented in the form of the product of the envelope times the cosinusoidal function. If we arrange the cosinusoidal factors in (6.12) according to the formulas for the sum and difference of the angles and use the expressions for trigonometric functions of multiple arguments in terms of the powers of these functions [13, p. 41], we get:

$$\left.\begin{aligned} r_{12}(\tau) &= e^{-2\pi a\tau c/(1+a)} \\ r_{21}(\tau) &= e^{-2\pi\tau c/(1+a)} \end{aligned}\right\}\frac{(n-1)!\,a^{n-0.5}}{2^{-2n+1}(2n-2)!\,(1+a)^{2n-1}(1+b^2)^n}\times$$

$$\times\left\{\sum_{l=0}^{n-1}\frac{l_{+}!\,(2\pi\tau c)^{l}}{l!\,l_{-}!\,(1+b^2)^{l}}\left[(1-C^2_{n+l}b^2+C^4_{n+l}b^4-\ldots)\times\right.\right.$$

$$\left.\left.\times\left\{\begin{matrix}\cos 2\pi f_2\tau - \\ \cos 2\pi f_1\tau +\end{matrix}\right\}(C^1_{n+l}\,b-C^3_{n+l}b^3+\ldots)\times\begin{matrix}\sin 2\pi f_2\tau \\ \sin 2\pi f_1\tau\end{matrix}\right]\right\},$$

where

$$C^n_m = \frac{m!}{(m-n)!\,n!}$$

According to this it is the envelope which is of the greatest interest, since the information contained in the "high-frequency" filling of the cross-correlation function has no practical use. Below analytical expressions are given for the envelopes of cross-correlation functions obtained for particular cases from these relationships.

1. Single-stage resonance amplifiers ($n = 1$)

$$\left.\begin{aligned} r^{or}_{12}(\tau) &= e^{-\frac{ax}{1+a}} \\ r^{or}_{21}(\tau) &= e^{-\frac{x}{1+a}} \end{aligned}\right\}\frac{2\sqrt{a}}{(1+a)\sqrt{1+b^2}},$$

where we have $x = 2\pi\tau c$.

Calculations made from these relationships for certain coefficient values of band a and for the shift in resonance frequency b are shown in Figs. 6.1 and 6.2. In these figures, as well as all subsequent figures, values of r_{12}^{or} lie to the right of axis $\tau = 0$, curve r_{21}^{or} is plotted to the left of this axis. It follows from the figures that the envelope

1) retains a maximal value at point $\tau = 0$ at any values of a and b;

2) has no derivative in the case of zero delay;

3) decreases in magnitude but does not change its shape when there is only a mutual displacement in the resonance frequencies of the amplifiers (a = 1, b = var);

4) becomes asymmetric with respect to zero delay when the pass bands of the linear amplifiers differ from one another (b = 0, a = var).

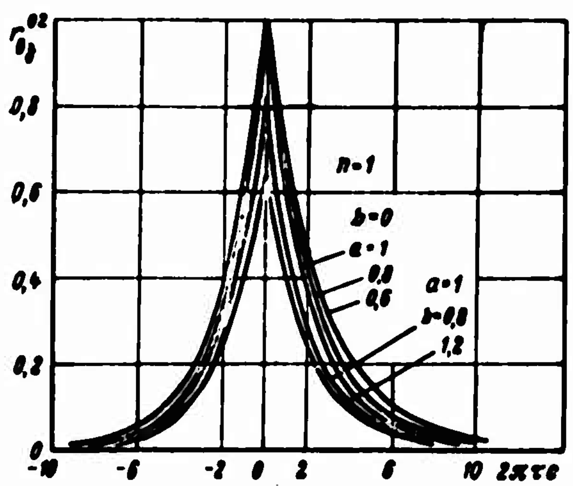

Fig. 6.1. Envelope of cross-correlation function (linear distortions created by single-stage resonance amplifiers).

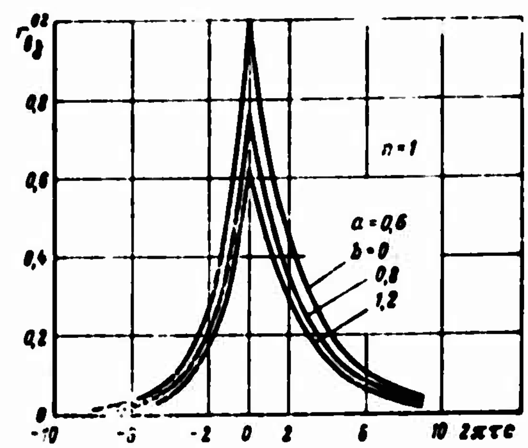

Fig. 6.2. Envelope of cross-correlation function (linear distortions created by single-stage resonance amplifiers).

2. Two-cascade resonance amplifiers (n = 2)

$$\left.\begin{matrix} r_{12}^{or}(\tau) \\ r_{21}^{or}(\tau) \end{matrix}\right\} = \left\{\begin{matrix} \exp\left[\frac{-ax}{1+a}\right] \\ \exp\left[\frac{-x}{1+a}\right] \end{matrix}\right\} \times$$

$$\times \frac{4\sqrt{a^3}\sqrt{x^2(1+b^2)+4x+4}}{(1+a)^3\sqrt{(1+b^2)^3}}.$$

In Figs. 6.3-6.8 the envelopes of the cross-correlation functions for n = 2-4 are plotted for certain coefficient values of band a and for the shift in resonance frequency b. Based on these graphic

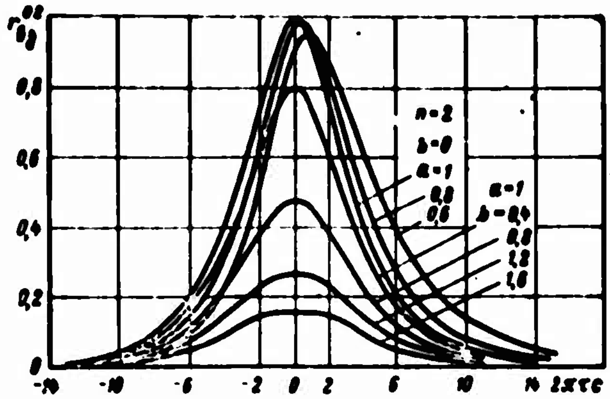

Fig. 6.3. Envelope of cross-correlation function (linear distortions created by two-stage resonance amplifiers).

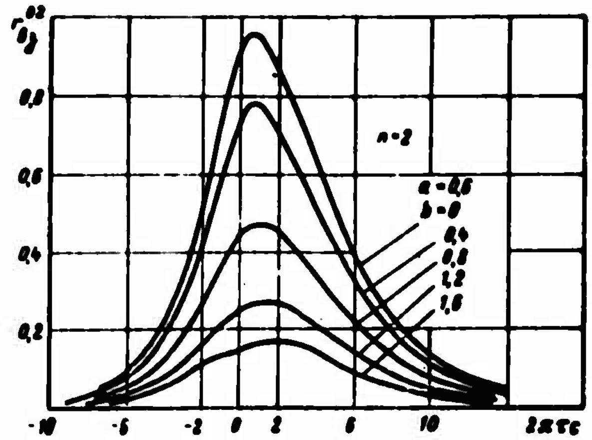

Fig. 6.4. Envelope of cross-correlation function (linear distortions created by two-stage resonance amplifiers).

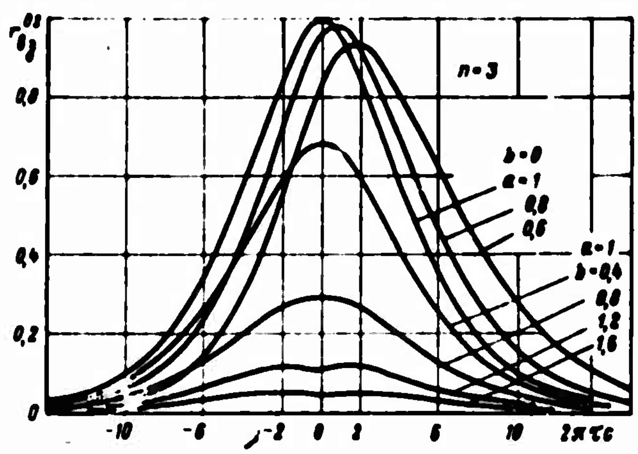

Fig. 6.5. Envelope of cross-correlation function (linear distortions created by three-stage resonance amplifiers).

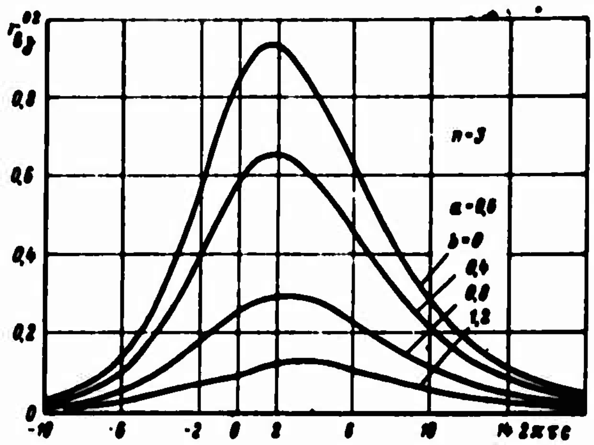

Fig. 6.6. Envelope of cross-correlation function (linear distortions created by three-stage resonance amplifiers).

Fig. 6.7. Envelope of cross-correlation function (linear distortions created by four-stage resonance amplifiers).

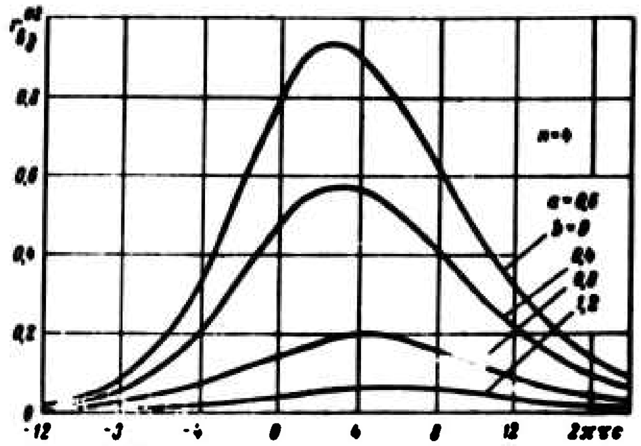

Fig. 6.8. Envelope of cross-correlation function (linear distortions created by four-stage resonance amplifiers).

dependences and the study of the envelopes the following conclusions may be drawn regarding the nature of the envelopes of the cross-correlation function when $n \geq 2$:

1) The envelope has a rounded peak at the point of maximum correlation;

2) the appearance of just one mutual shift in the resonance frequencies leads to a decrease in the values of the cross-correlation function and to a considerable "flattening" in the curve in the region of maximum correlation. In the case of significant maladjustment ($b > 1$) the envelope has a slight two-humped nature;

3) if the pass bands are nonidentical ($a \neq 1$), this leads to a shift in the maximum of the envelope along the X-axis and to its asymmetry relative to the coordinate maximum.

4) At any values of nonidentical pass band coefficient a or resonance frequency shift coefficient b within the limits of $-1 \leq b \leq 1$ the envelope has a single peak and falls asymptotically to zero as delay increases. These graphic dependences have been

plotted for b > 0 and a < 1. This was determined by the fact that the envelope of cross-correlation function, as we have learned from the above relationships, is the mirror [image] of the Y axis with a 1/a change in a and does not depend on the sign of the coefficient of shift in resonance frequency b.

6.4. EFFECT OF NONIDENTICAL CHARACTERISTICS IN LINEAR UNITS ON ACCURACY OF MEASURING DELAY AND OUTPUT SIGNAL/NOISE RATIO

The shift in the maximal correlation point, which is caused by nonidentical characteristics in the linear units of a correlation system, leads to systematic error in measuring the delay of noise signals in channels. As we learned from the materials of the preceding section, if the number of cascades $n \geq 2$ and the pass band of the filter Φ_1 is greater than pass band Φ_2, then the maximum of the envelope will be observed at a certain additional delay τ_M in channel Φ_1. The magnitude of additional delay can be found from the graphic dependence of Fig. 6.9, which shows the systematic error of measuring delay as a function of the coefficient of nonidentity of a and b. When a > 1 the maximum of the envelope is observed at a certain additional delay in the amplification channel Φ_2. The magnitude of additional delay may be found by means of the same graph, although in this case we must use the reverse quantity for the coefficient of nonidentity of the pass band. As already mentioned, when n = 1, there is no systematic error. This is a "degenerate" case, as it were.

Determining the signal/noise ratio at the output of the averaging filter [formula (6.5)] involves calculating the integral

$$\int_{-\infty}^{\infty} r_1(\tau) r_2(\tau) d\tau,$$

where $r_1(\tau)$ and $r_2(\tau)$ are the normalized autocorrelation functions of the noise processes at the outputs of Φ_1 and Φ_2, respectively.

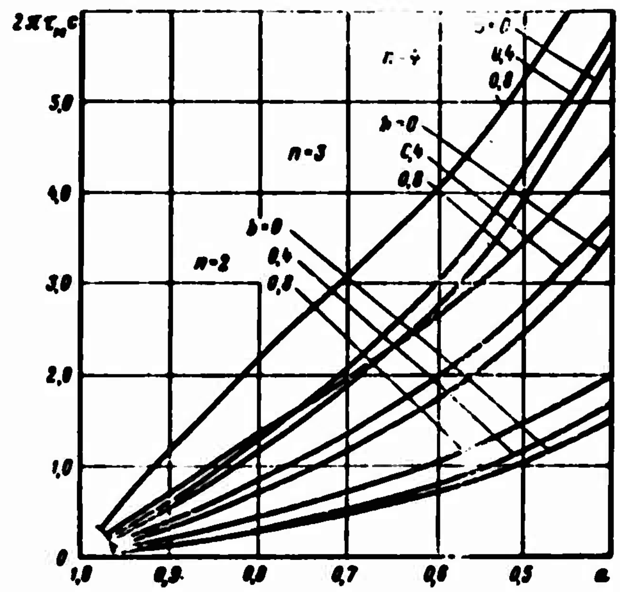

Fig. 6.9. Systematic error of measuring delay.

By means of relationship (6.13) we get

$$\int_{-\infty}^{\infty} r_1(\tau)\, r_2(\tau)\, d\tau = \left[\frac{(n-1)!}{(2n-2)!}\right]^2 \times$$
$$\times \sum_{l=0}^{n-1} \sum_{k=0}^{n-1} \frac{l_+!\, k_+!}{l!\, l_-!\, k!\, k_-!} \times$$
$$\times \int_{-\infty}^{\infty} e^{-\pi|\tau|(B_1+B_2)} (2\pi|\tau| B_1)^{l_-} (2\pi|\tau| B_2)^{k_-}\, 0.5 \times$$
$$\times [\cos 2\pi(f_2 - f_1)\tau + \cos 2\pi(f_2 + f_1)\tau]\, d\tau,$$

where $k_- = n - k - 1$, $k_+ = n + k - 1$.

In integrating we ignore the participation of the second term in the integrand, since in view of the narrow-band nature of the linear amplifiers $f_1 + f_2 \gg B_1 + B_2$. In the integral we exchange the variable $z = \pi\tau(B_1 + B_2)$. Then, considering that the integrand function is even and preserving the system of coefficient designations of (6.11), by means of the table integral [13, p. 504] we get

$$\int_{-\infty}^{\infty} r_1(\tau) r_2(\tau)\, d\tau = \frac{1}{2nc}\left[\frac{(n-1)!}{(2n-2)!}\right]^2 \times$$

$$\times \sum_{l=0}^{n-1}\sum_{k=0}^{n-1} \frac{l_+!\, k_+!}{l!\, l_-!\, k!\, k_-!} \left(\frac{2}{1+a}\right)^{l_-} \left(\frac{2a}{1+a}\right)^{k_-} \times$$

$$\times \frac{(p_{kl}-2)!}{(1+b)^{\frac{p_{kl}-1}{2}}} \cos\left[(p_{kl}-1)\operatorname{arctg} b\right]. \qquad (6.14)$$

where $p_{kl} = 2n - k - l$.

The signal/noise ratio at the output of the correlation system in the case of maximal correlation and identical characteristics in the linear unit (i.e., when $B_1 = B_2 = c$, $a = 1$, $b = 0$ and $r_{вз} = 1$) will be equal on the basis of (6.5) and (6.14) to

$$\left(\frac{P_c}{P_ш}\right)_{вх} = \left(\frac{S_c}{S_ш}\right)^2 \frac{2nc}{\left(1 + 2\frac{S_c}{S_ш}\right) B_y \xi}, \qquad (6.15)$$

where

$$\xi = \left[\frac{(n-1)!}{(2n-2)!}\right]^2 \sum_{l=0}^{n-1}\sum_{k=0}^{n-1} \frac{l_+!\, k_+!\, (p_{kl}-2)!}{l!\, l_-!\, k!\, k_-!}.$$

From this expression it follows that the signal/noise ratio at the system output is proportional to the ratio of mean arithmetic band pass c to the effective band of the averaging filter B_y and the square of the signal/noise ratio at the input (at low input signals when $S_c \ll S_ш$). Coefficient ξ in (6.15) takes into account, depending on the number of stages, the narrowing in the band pass of an n-stage resonance amplifier as compared to a single-stage amplifier.

The signal/noise ratio at the output of an averaging filter in the case of nonidentical parameters in the linear amplifiers will be represented as an analogous characteristic in the case of total identity (6.15) and as a certain loss coefficient κ as follows:

$$\frac{P_c}{P_ш} = \varkappa \left(\frac{P_c}{P_ш}\right)_{ид}. \qquad (6.16)$$

Loss coefficient κ can be determined from equality (6.16) if we consider (6.5), (6.14), and (6.15)

$$\varkappa = \frac{r^2_{вз} \sum\limits_{l=0}^{n-1} \sum\limits_{k=0}^{n-1} \frac{l_+!\,k_+!}{l!\,k!\,l_-!\,k_-!} \times}{\sum\limits_{l=0}^{n-1} \sum\limits_{k=0}^{n-1} \frac{l_+!\,k_+!}{l!\,k!\,l_-!\,k_-!} \left(\frac{2a}{1+a}\right)^{k_-} \left(\frac{2}{1+a}\right)^{l} \times} \rightarrow$$

$$\rightarrow \frac{\times\,(p_{h1}-2)!}{\times \frac{(p_{h1}-2)!}{(1+b^2)^{(p_{h1}-1)/2}} \cos\left[(p_{h1}-1)\,\mathrm{arctg}\, b\right]}. \qquad (6.17)$$

Note that loss coefficient κ depends on the value of the normalized cross-correlation function $r^2_{вз}$, which is determined by delay in the noise signal of one channel relative to another. Detecting the signal and measuring delay are usually done at a maximum value of the function $r_{вз}$ or in passing through the maximum. At this point it makes sense to find the signal/noise ratio. For this case Figs. 6.10 and 6.11 show the graphic dependence of loss coefficient κ on the number of stages n and the identity coefficients a and b. From the figures it follows that losses in the signal/noise ratio are not great, even when there is considerable nonidentity in the parameters of the linear units of the correlation system.

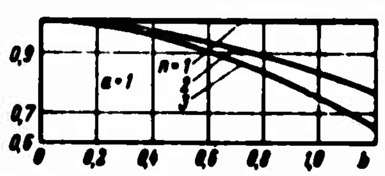

Fig. 6.10.

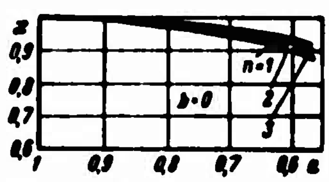

Fig. 6.11.

Fig. 6.10. Deterioration in signal/ noise ratio in the case of linear distortions in the studied processes.

Fig. 6.11. Deterioration in signal/ noise ratio in the case of linear distortions in the studied processes.

The obtained relationships can be used to determine the requirements of linear units in correlation systems and to analyze measurement error.

6.5. MEASURING CORRELATION FUNCTION IN THE CASE OF DISTORTIONS IN ONE CHANNEL

Let us examine the case of a correlation system with a filter in one of the channels of the correlator (block diagram of Figs. 3.3 and 3.8) which has a transmission coefficient identically equal to one. In this case only the filter in the second channel will affect measuring the correlation function of the input process. Such operational conditions in the first approximation may occur in correlation systems with memory [storage] such as, for example, an active noise radar set. The recorded signal in this case has great intensity and is not subjected to substantial transformations in the channel prior to the correlator.

We will assume that the studied random process has a spectral density with a shape analogous to the frequency characteristic of a simple oscillatory contour with a resonance frequency of $f_{ц}$ and a bandwidth of B_{c}, i.e.,

$$G(f)=\frac{2}{\pi B_c}\left\{1+\left[\frac{2(f-f_{ц})}{B_c}\right]^2\right\}^{-1};$$

$$R(\tau)=e^{-\pi|\tau|B_c}\cos 2\pi f_{ц}\tau.$$

Let us examine the error of correlation measurements for a case where filter Φ_2 has the selectivity of a simple oscillatory circuit with resonance frequency $f_{ц}-f_{сд}$ and band pass $B_{и}$. Quantity $f_{сд}$ represents a shift in the tuning frequency of the resonance amplifier in relation to the middle frequency of the signal spectrum. The transmission coefficient of the filter is determined by the relationships

$$K_{\text{н}}(f)=\left\{1+\left[\frac{2(f-f_{\text{ц}}+f_{\text{сд}})}{B_{\text{н}}}\right]^2\right\}^{-1/2},$$

$$\varphi_{\text{н}}(f)=\operatorname{arctg}\frac{2(f-f_{\text{ц}}+f_{\text{сд}})}{B_{\text{н}}}.$$

If we introduce the original data into formula (6.3) and designate $B_{\text{н}}=a_{\text{н}}B_{\text{c}}$, $f_{\text{сд}}=b_{\text{н}}\frac{B_{\text{c}}}{2}$, then after integrating, for the normalized correlation function we get the expression

$$\begin{aligned}r_{21}&=\frac{K_{\text{н}}a_{\text{н}}}{L}\{[2b_{\text{н}}^2+(a_{\text{н}}-1)\Pi]\,e^{-\pi\tau B_{\text{c}}}\times\\&\times\cos 2\pi f_{\text{ц}}\tau+[2b_{\text{н}}(a_{\text{н}}-1)-b_{\text{н}}\Pi]\,e^{-\pi\tau B_{\text{c}}}\sin 2\pi f_{\text{ц}}\tau\},\end{aligned}\qquad(6.18)$$

$$\begin{aligned}r_{12}&=\frac{K_{\text{н}}a_{\text{н}}}{L}\{[(a_{\text{н}}+1)\Pi-2b_{\text{н}}^2]\,e^{-\pi\tau B_{\text{c}}}\times\\&\times\cos 2\pi f_{\text{ц}}\tau+[2b_{\text{н}}(a_{\text{н}}+1)+b_{\text{н}}\Pi]\,e^{-\pi\tau B_{\text{c}}}\times\\&\times\sin 2\pi f_{\text{ц}}\tau+2(2b_{\text{н}}^2-\Pi)\,e^{-\pi\tau a_{\text{н}}B_{\text{c}}}\times\\&\times\cos 2\pi(f_{\text{ц}}-f_{\text{сд}})\tau-4b_{\text{н}}a_{\text{н}}e^{-\pi\tau a_{\text{н}}B_{\text{c}}}\times\\&\times\sin 2\pi(f_{\text{ц}}-f_{\text{сд}})\tau\}.\end{aligned}$$

where we have $\Pi=b_{\text{н}}^2+a_{\text{н}}^2-1$, $L=4b_{\text{н}}^2+(b_{\text{н}}^2+a_{\text{н}}^2-1)^2$. Quantity $K_{\text{н}}$ is a normalized factor which considers the deformation of the energy spectrum of the studied voltage by filter Φ_2. This coefficient is equal to

$$K_{\text{н}}=\sqrt{\frac{1}{a_{\text{н}}[2b_{\text{н}}^2+(a_{\text{н}}-1)\Pi]}}.$$

Let us analyze the individual cases of distortion under the condition that the frequency shift $f_{\text{сд}}=0$. Then

$$K_{\text{н}}=\sqrt{\frac{a_{\text{н}}+1}{a_{\text{н}}}},\qquad(6.19)$$

$$r_{21}=\sqrt{\frac{a_{\text{н}}}{a_{\text{н}}+1}}\,e^{-\pi\tau B_{\text{c}}}\cos 2\pi f_{\text{ц}}\tau,$$

$$r_{12}=r_{21}\frac{2e^{-\pi\tau(a_{\text{н}}-1)B_{\text{c}}}-a_{\text{н}}-1}{1-a_{\text{н}}}.\qquad(6.20)$$

Analysis shows that the envelope maximum of the cross-correlation function is observed at a certain additional delay $\tau_{\text{м}}$ in channel Φ_1, which is determined by the relationship

$$\tau_{\text{м}}=\frac{1}{\pi B_{\text{э}}(1-a_{\text{и}})}\ln\frac{1+a_{\text{и}}}{2a_{\text{и}}}.$$

In correlation measurements it is important to know how voltage changes at the correlator output depending on maladjustment of the distorting filter (i.e., the nonnormalized correlation function). Let us analyze a case where the normalizing coefficient is determined by (6.19), i.e., does not depend on the coefficient of maladjustment $b_{\text{и}}$, while the band coefficient $a_{\text{и}} = 1$. From (6.18) we find

$$r_{21}=\sqrt{\frac{2}{4+b_{\text{и}}^2}}\,e^{-\pi\tau B_{\text{э}}}\cos(2\pi f_{\text{ц}}\tau+\vartheta_1),$$

$$r_{12}=\frac{\sqrt{2}}{b_{\text{и}}\sqrt{4+b_{\text{и}}^2}}\,e^{-\pi\tau B_{\text{э}}}\times$$

$$\times\sqrt{4+(4+b_{\text{и}}^2)-4\sqrt{4+b_{\text{и}}^2}\cos(2\pi f_{\text{сд}}\tau+\vartheta_1)}\times$$

$$\times\cos(2\pi f_{\text{ц}}\tau-\vartheta_2). \qquad (6.21)$$

where

$$\vartheta_1=\operatorname{arctg}\frac{b_{\text{и}}}{2},$$

$$\vartheta_2=\operatorname{arctg}\frac{4+b_{\text{и}}^2+2b_{\text{и}}\sin 2\pi f_{\text{сд}}\tau-4\cos 2\pi f_{\text{сд}}\tau}{2b_{\text{и}}\cos 2\pi f_{\text{сд}}\tau+4\sin 2\pi f_{\text{сд}}\tau}.$$

By aralyzing relationships (6.20) and (6.21) we find that if there is a distorting filter in one channel of the correlation system the following takes place:

1) shift in maximum of output voltage along axis τ, which occurs as a result of the group delay time of the distorting filter, and a decrease in the magnitude of the maximum, which can be explained by the nonlinearity of the phase characteristic;

2) the envelope of the correlation function is asymmetric relative to the maximum; the asymmetry becomes noticeable when the band of the distorting filter is equal or is narrower than the band of the energy spectrum of the studied process;

3) there is a phase shift in the "high-frequency filling" of the envelope of the correlation function.

In the particular case, which is characterized by the relationships of (6.21), we find that for r_{21} phase shift ϑ_1 depends only on the magnitude of maladjustment and does not depend on τ. For r_{12} phase ϑ_2 depends on maladjustment $f_{сд}$ and for different values of τ it has a different value. Figures 6.12 and 6.13 show the dependence of the envelope of the correlation function on the dimensionless quantity $x = \pi\tau B_c$ for the two particular cases analyzed. The experimental values were obtained by means of a difference frequency correlator, whose simplified system is shown in Fig. 2.5. Deviation of experimental points from theoretical points resulted primarily from the additional linear distortions introduced by the delay line.

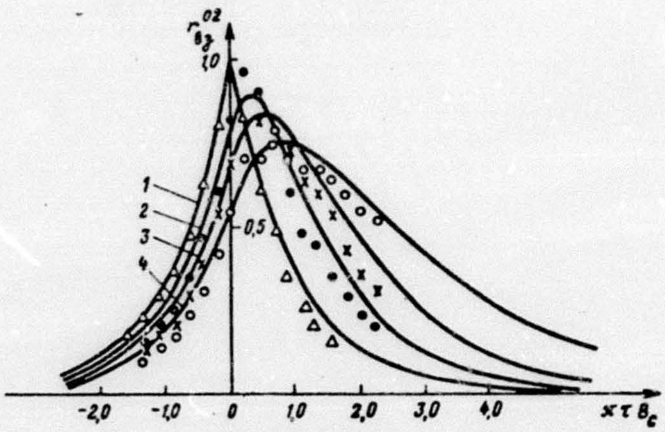

Fig. 6.12. Envelope of cross-correlation functions in the case of linear distortions in one channel: 1 - envelope of autocorrelation function of single oscillatory circuit; 2, 3, 4 - envelope of correlation function for parameters a_u = 2; 1; 0; 5; b_u = 0.

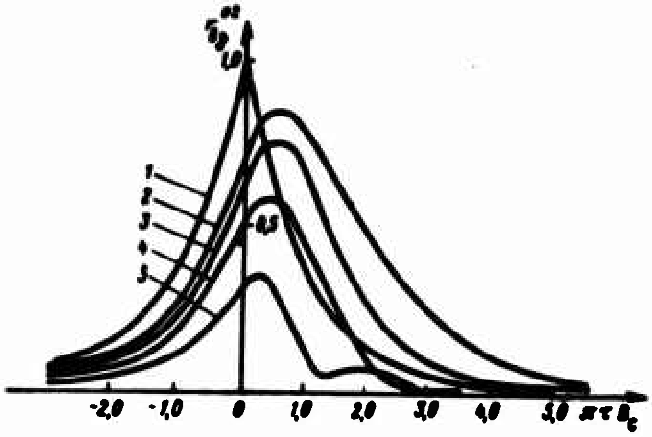

Fig. 6.13. Envelope of cross-correlation function in the case of linear distortions in one channel: 1 - envelope of autocorrelation function of single o cillatory circuit, no distortion; 2, 3, 4, 5 - envelope of correlation function for coefficients b_u = 0; 1; 2; 4; a_u = 1.

SEVENTH CHAPTER

CORRELATION PROPERTIES OF FILTERED PHASE-MANIPULATED SIGNALS

7.1. AUTOCORRELATION FUNCTION

Linear distortions in correlation systems can be caused not only by filters in the correlator channels, but also by the general filter Φ_0 (see Chapter 3). Distortions of this type occur in systems in which the emitted and reference signals are formed by means of a single code source.

Our definition of a phase-manipulated signal is a high-frequency oscillation with a frequency of f_c, whose phase is manipulated by 0 and π according to the law of change in symbols of a binary pseudorandom sequence (see § 1.3).

In this chapter we estimate the effect of linear filtration on the correlation properties of phase-manipulated signals. We study cases in which the frequency of the carrier is great as compared to the width of the energy spectrum of the sequence.

The correlation function of a periodic binary sequence is determined by the relationship [12, 25]

$$r_{\text{п}}(\tau)=\begin{cases} 1-\frac{|\tau|}{t_0}\left(\frac{N+1}{N}\right) & \text{при } |\tau|<t_0, \\ -\frac{1}{N} & \text{при } t_0<|\tau|<0.5Nt_0. \end{cases} \qquad (7.1)$$

[при = when]

while the energy spectrum can be represented in the form of delta-functions with weight coefficients, which are found from the Fourier transform by autocorrelation function (7.1). If we perform the calculations, we find

$$S_n(f) = \sum_{n=-\infty}^{\infty} \frac{N+1}{N^2}\left(\frac{\sin\frac{n\pi}{N}}{\frac{n\pi}{N}}\right)^2 \delta(f - nf_u) - \frac{1}{N}\delta(f). \tag{7.2}$$

where $f_u = 1/Nt_0$ is the repetition frequency of the sequence.

From this expression we find that the envelope of the energy spectrum vanishes when $n = \pm N, \pm 2N, \ldots$. Then, the width of the spectrum with respect to the first zeros of the envelope is

$$B_u = 2Nf_u = \frac{2}{t_0} = 2f_{\text{такт}},$$

where $f_{\text{такт}}$ is the frequency of the clock pulses of the sequences. The energy spectrum of a signal which is phase-manipulated by means of a periodic sequence is determined by the relationship

$$S_c(f) = 0{,}5[S_n(f - f_c) + S_n(f + f_c)]. \tag{7.3}$$

The autocorrelation function of a pseudorandom signal passing through a linear filter with an amplitude-frequency characteristic $K(f)$ is found as the Wiener-Khinchin transform of its energy spectrum at the output:

$$R_\phi = \frac{1}{k_н}\int_{-\infty}^{\infty} S_c(f) K^2(f) e^{i2\pi f\tau} df,$$

where $k_н$ is the normalizing coefficient.

If here we substitute (7.3), then after replacing the variables in the integral and transformations we get

$$R_\Phi = \frac{1}{k_{\text{н}}}\left\{\cos 2\pi f_c\tau \int\limits_{-\infty}^{\infty} S_u(f) K^2(f+f_c)\cos 2\pi f\tau df - \right.$$

$$\left. - \sin 2\pi f_c\tau \int\limits_{-\infty}^{\infty} S_u(f) K^2(f+f_c)\sin 2\pi f\tau df\right\}. \qquad (7.4)$$

Let us examine the simpler case where the frequency characteristic of the linear filter is symmetrical relative to its central frequency, which is equal to the carrier frequency of the pseudorandom signal f_c. Then, from (7.4) we find

$$R_\Phi(\tau) = r_\Phi(\tau)\cos 2\pi f_c\tau,$$

where

$$r_\Phi(\tau) = \frac{1}{k_{\text{н}}}\int\limits_{-\infty}^{\infty} S_n(f) K^2(f+f_c)\cos 2\pi f\tau df. \qquad (7.5)$$

Let us substitute in the expression for the envelope of the correlation function of the phase-manipulated signal (7.5) the relationship of (7.2) for the energy spectrum of the sequence. If we use the property of the delta-functions in calculating the integral, then after trigonometric transformations we get

$$r_\Phi(\tau) = \frac{1}{k_{\text{н}}}\left\{\frac{K^2(f_c)}{N^2} + \frac{N+1}{\pi^2}\sum_{n=1}^{\infty} \times\right.$$

$$\times \frac{K^2\left(\frac{n}{Nt_0}+f_c\right)}{n^2}\left[\cos\frac{2\pi n\theta}{N} \quad 0{,}5\cos\frac{2\pi n}{N}(1+\theta) - \right.$$

$$\left.\left. - 0{,}5\cos\frac{2\pi n}{N}(1-\theta)\right]\right\}. \qquad (7.6)$$

where $\theta = \tau/t_0$ is relative delay time.

From this relationship when $K\left(\frac{n}{Nt_0}+f_c\right) = 1$ it follows that the correlation function of an undistorted pseudorandom sequence is expressed as the series

$$r_u(\theta)=\frac{1}{N^2}+\frac{N+1}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\left[\cos\frac{2\pi n}{N}\theta-\right.$$

$$\left.-0,5\cos\frac{2\pi n}{N}(1+\theta)-0,5\cos\frac{2\pi n}{N}(1-\theta)\right]. \tag{7.7}$$

which by means of the series [16, series VB-1]

$$\sum_{n=1}^{\infty}\frac{\cos nx}{n^2+a^2}=\frac{\pi\,\mathrm{ch}\,(ax-a\pi)}{2a\,\mathrm{sh}\,a\pi}-\frac{1}{2a^2}$$

is reduced to the expression

$$r_u(\theta)=\begin{cases}1-|\theta|\frac{N+1}{N} & \text{при } |\theta|<1,\\ -\frac{1}{N} & \text{при } 1<|\theta|<0,5(N-1).\end{cases} \tag{7.8}$$

If we consider the symbols which we have introduced, then relationship (7.8), as one might expect, coincides with (7.1). When calculating (7.6) it is convenient to use the function

$$\psi(\theta)=\sum_{n=1}^{\infty}\frac{K^2\left(\frac{n}{K\tau_0}+l_0\right)}{n^2}\cos\frac{2\pi n}{N}\theta. \tag{7.9}$$

Function (7.9) contains only cosinusoidal terms and, consequently, it is even and periodic with a period of $\theta_{\text{пер}} = N$. Then, from (7.6) we find that the envelope has the following properties

$$r_\Phi(\theta)=r_\Phi(-\theta),\quad r_\Phi(\theta+N)=r_\Phi(\theta),$$
$$r_\Phi\left(\theta+\frac{N}{2}\right)=r_\Phi\left(\theta-\frac{N}{2}\right).$$

Thus, we are studying the envelope only on section $0 \leq \theta \leq N/2$.

In the overwhelming majority of cases of synthesizing series [16] the analytical expression $\psi(\theta)$ in a closed form can be used

successfully only on section $0 \leq \theta \leq N/2$ in view of the difficulty of writing this function with a single analytical expression for the entire axis θ. Therefore, the envelope of the autocorrelation function of the filtered phase-manipulated signal of (7.6) can be represented as:

$$r_{\Phi} = \frac{K^2(f_0)}{k_n N^2} + \frac{N+1}{k_n \pi^2} [\psi(\theta) - 0.5\psi(\theta+1) - \\ - 0.5\psi(|1-\theta|)] \quad (7.10)$$

when $0 \leq \theta \leq N/2$.

7.2. PARTICULAR CASES OF SIGNAL FILTRATION

Let us examine the influence of the passband of several specific types of filters on the autocorrelation function of a filtered phase-manipulated signal, whose carrier frequency coincides with the central frequency of the filter.

1. Filter with Single Oscillatory Circuit

The amplitude-frequency characteristic of such a linear filter is determined by the expression

$$K(f) = \left\{1 + \left[\frac{2(f - f_0)}{B}\right]^2\right\}^{-1/2},$$

where B is the passband at a level of -3 dB. The square of the amplitude-frequency characteristic is represented in a form which can be easily studied

$$K^2\left(\frac{n}{N\tau_0} + f_0\right) = \left(1 + \frac{n^2}{a^2 N^2}\right)^{-1}, \quad (7.11)$$

where

$$a = \frac{Bt_0}{2} = \frac{B}{2f_{\text{такт}}}.$$

The dimensionless coefficient α binds the parameters of the filter and the pseudorandom sequence. It is equal to the ratio of half of the passband of the filter to the clock frequency of the pseudorandom sequence. Let us substitute (7.11) in (7.6) and break down the rational number which appears here into simple terms.

If we group the terms, then for the envelope of the correlation function of a phase-manipulated signal filtered by a single oscillatory circuit, we get the following expression:

$$\begin{aligned} r_{\text{ок}}(\theta) = {} & \frac{1}{k_{\text{н}}^{\text{ок}} N^2} + \frac{N+1}{k_{\text{н}}^{\text{ок}} \pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \Big[\cos\frac{2\pi n}{N}\theta - \\ & - 0{,}5 \cos\frac{2\pi n}{N}(1+\theta) - 0{,}5 \cos\frac{2\pi n}{N}(1-\theta) \Big] - \\ & - \frac{N+1}{k_{\text{н}}^{\text{ок}} \pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2 + a^2 N^2} \Big[\cos\frac{2\pi n}{N}\theta - 0{,}5 \cos\frac{2\pi n}{N}(1+\theta) - \\ & - 0{,}5 \cos\frac{2\pi n}{N}(1-\theta) \Big]. \end{aligned}$$

By means of formula (7.7) this expression can be represented in the form of

$$r_{\text{ок}}(\theta) = \frac{r_{\text{н}}(\theta) - \rho_{\text{ок}}(\theta)}{k_{\text{н}}^{\text{ок}}}, \qquad (7.12)$$

where the normalizing coefficient $k_{\text{н}}^{\text{ок}}$, which is determined from the condition $r_{\text{ок}}(0) = 1$, is equal to

$$k_{\text{н}}^{\text{ок}} = 1 - \rho_{\text{ок}}(0). \qquad (7.13)$$

Function $\rho_{\text{ок}}(\theta)$, which indicates the degree of the effect of the single oscillatory circuit, is equal to

$$\rho_{ок}(\theta)=\frac{N+1}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2+\alpha^2N^2}\left[\cos\frac{2\pi n}{N}\theta-\right.$$

$$\left.-0.5\cos\frac{2\pi n}{N}(1+\theta)-0.5\cos\frac{2\pi n}{N}(1-\theta)\right].$$

We use a series [16, VB-1] to simplify the obtained expressions.

Let us also consider that coefficient α in practical systems has an order of one, while the length of the sequence has an order of hundreds and thousands. When $\alpha N\pi > 5$, which occurs in the majority of cases, we can write the following approximate relationships (error does not exceed 0.005%):

$$\operatorname{cth}\alpha N\pi\approx 1,$$
$$\operatorname{sh}2\pi\alpha-\operatorname{cth}\alpha N\pi\, 2\operatorname{sh}^2\pi\alpha\approx 1-e^{-2\pi\alpha}.$$

Considering these approximations, we get expressions for the function $\rho_{ок}(\theta)$:

a) $0 \leq \theta \leq 1$

$$\rho_{ок}(\theta)=\frac{N+1}{2\alpha N\pi}\left(e^{-2\pi\alpha\theta}-e^{-2\pi\alpha}\operatorname{ch}2\pi\alpha\theta\right). \quad (7.14)$$

b) $1 \leq \theta \leq N/2$

$$\rho_{ок}(\theta)=-\frac{N+1}{\alpha N\pi}\operatorname{sh}^2\pi\alpha e^{-2\pi\alpha\theta}. \quad (7.15)$$

The envelope of the autocorrelation function of a filtered phase-manipulated signal for a sequence length of N = 127 has been plotted in Fig. 7.1 for several values of band coefficient α. Shown for comparison is the autocorrelation function of the undistorted binary pseudorandom sequence of (7.8). These dependences indicate that the envelope:

1) has a very rounded peak at the point of maximal correlation;

2) is expanded along the X-axis (the lower the value of α, the greater this expansion);

3) monotonically falls with an increase in θ toward the established limit.

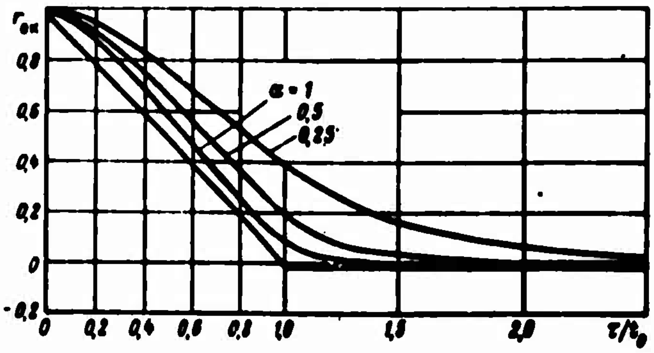

Fig. 7.1. Envelope of correlation function of phase-manipulated signal which has passed through a single oscillatory circuit.

2. Bandpass Filter of System of Two Identically Connected Circuits

The amplitude-frequency characteristic of a system with two identical connected circuits is determined by the expression

$$K(f) = \left\{\left[\frac{2(f-f_0)}{B}\right]^4 + 2(1-p^2)\times\right.$$
$$\left.\times\left[\frac{2(f-f_0)}{B}\right]^2 + (1+p^2)^2\right\}^{-\frac{1}{2}},$$

where B is the passband of each circuit of the filter on a level of -3 dB; p is the coupling factor.

The square of the amplitude-frequency characteristic is written in the form

$$K^2\left(\frac{n}{Nt_0}+f_0\right)=\left[\left(\frac{n}{\alpha N}\right)^4+\right.$$

$$\left.+2(1-p^2)\left(\frac{n}{\alpha N}\right)^2+(1+p^2)^2\right]^{-1}. \qquad (7.16)$$

Coefficient α is determined just as in the preceding case [formula (7.11)].

Expression (7.16) is substituted in (7.6). If we expand the rational part into simple factors and group the terms with (7.7) considered, then for the envelope of the autocorrelation function the relationship

$$r_{сн}(\theta)=\frac{r_н(\theta)-\rho_{сн}(\theta)}{k_н^{сн}}, \qquad (7.17)$$

where the normalizing coefficient is equal to

$$k_н^{сн}=1-\rho_{сн}(0), \qquad (7.18)$$

while function $\rho_{сн}(\theta)$, which indicates the degree of influence of the bandpass filter, is determined by the expression

$$\rho_{сн}(\theta)=\frac{N+1}{\pi^2}\sum_{n=1}^{\infty}\frac{n^2+2(\alpha N)^2(1-p^2)}{n^4+2(\alpha N)^2(1-p^2)n^2+(1+p^2)(\alpha N)^4}\times$$

$$\times\left[\cos\frac{2\pi n}{N}\theta-0{,}5\cos\frac{2\pi n}{N}(1+\theta)-\right.$$

$$\left.-0{,}5\cos\frac{2\pi n}{N}(1-\theta)\right]. \qquad (7.19)$$

In order to synthesize the series (7.19) let us examine the following function:

$$\psi(\theta)=\sum_{n=1}^{\infty}\frac{n^2+2(\alpha N)^2(1-p^2)}{n^4+2(\alpha N)^2(1-p^2)n^2+(1+p^2)(\alpha N)^4}\times$$

$$\times\cos\frac{2\pi n}{N}\theta.$$

We will use series [16, VK-1, DZh-1] in the following form:

$$\sum_{n=1}^{\infty} \frac{n^2 \cos nx}{(n^2+c^2)(n^2+d^2)} = \frac{\pi}{2(c^2-d^2)} \times$$
$$\times \left[\frac{c \operatorname{ch} c(x-\pi)}{\operatorname{sh} c\pi} - \frac{d \operatorname{ch} d(x-\pi)}{\operatorname{sh} d\pi}\right],$$
$$\sum_{n=1}^{\infty} \frac{\cos nx}{(n^2+c^2)(n^2+d^2)} = \frac{1}{c^2-d^2}\left[\frac{1}{2c^2} - \frac{1}{2d^2} - \right.$$
$$\left. - \frac{\pi \operatorname{ch} c(x-\pi)}{2c \operatorname{sh} c\pi} + \frac{\pi \operatorname{ch} d(x-\pi)}{2d \operatorname{sh} d\pi}\right].$$

By means of these series function $\psi(\theta)$ after certain transformations can be written in the form of

$$\psi(\theta) = \frac{\pi}{4\alpha N p(1+p^2)(\operatorname{ch} 2\alpha N\pi - \cos 2\alpha N p\pi)} \times$$
$$\times \{(3-p^2)p[\operatorname{sh}(2\alpha N\pi - 2\pi\alpha\theta)\cos 2\pi\alpha p\theta +$$
$$+ \operatorname{sh} 2\pi\alpha\theta \cos(2\alpha N p\pi - 2\pi\alpha p\theta)] + (1-3p^2) \times$$
$$\times [\operatorname{ch}(2\alpha N\pi - 2\pi\alpha\theta)\sin 2\pi\alpha p\theta + \operatorname{ch} 2\pi\alpha\theta \times$$
$$\times \sin(2\alpha N p\pi - 2\pi\alpha p\theta)]\}.$$

Although the obtained expression is accurate, it is nevertheless awkward. In cases of practical interest $\alpha N\pi > 10$. Thus, we can use the following approximate formulas, which are valid when $0 \leq \theta \leq N/2$:

$$\operatorname{ch} 2\alpha N p\pi - \cos 2\alpha N p\pi \approx 0{,}5e^{2\alpha N p\pi},$$
$$\operatorname{sh}(2\alpha N\pi - 2\pi\alpha\theta) \approx \operatorname{ch}(2\alpha N\pi - 2\pi\alpha\theta) \approx$$
$$\approx 0{,}5e^{2\alpha N\pi - 2\pi\alpha\theta}.$$

Considering these approximations for function $\psi(\theta)$ we get the following expression:

$$\psi(\theta) = \frac{\pi}{4\alpha N p(1+p^2)}[(3-p^2)p e^{-2\pi\alpha\theta}\cos 2\pi\alpha p\theta +$$
$$+ (1-3p^2)e^{-2\pi\alpha\theta}\sin 2\pi\alpha p\theta].$$

while the series (7.19) for the two intervals of change in θ are written as follows:

a) $0 \leq \theta \leq 1$

$$
\begin{aligned}
\rho_{ck}(\theta) = {} & \frac{N+1}{4\pi\alpha N(1+p^2)} \Big\{ e^{-2\pi\alpha\theta} \cos 2\pi\alpha p\theta \big[(3-p^2) \times \\
& \times (1 - 0{,}5e^{-2\pi\alpha} \cos 2\pi\alpha p) - \frac{1-3p^2}{2p} e^{-2\pi\alpha} \sin 2\pi\alpha p\big] + \\
& + e^{-2\pi\alpha\theta} \sin 2\pi\alpha p\theta \Big[\frac{3-p^2}{2} e^{-2\pi\alpha} \sin 2\pi\alpha p + \\
& + \frac{1-3p^2}{p} (1 - 0{,}5e^{-2\pi\alpha} \cos 2\pi\alpha p)\Big] - \\
& - e^{2\pi\alpha\theta} \cos 2\pi\alpha p\theta \Big[\frac{3-p^2}{2} e^{-2\pi\alpha} \cos 2\pi\alpha p + \\
& + \frac{1-3p^2}{2p} e^{-2\pi\alpha} \sin 2\pi\alpha p\Big] - e^{2\pi\alpha\theta} \sin 2\pi\alpha p\theta \times \\
& \times \Big[\frac{3-p^2}{2} e^{-2\pi\alpha} \sin 2\pi\alpha p - \frac{1-3p^2}{2p} e^{-2\pi\alpha} \cos 2\pi\alpha p\Big]\Big\}. \qquad (7.20)
\end{aligned}
$$

b) $1 \leq \theta \leq N/2$

$$
\begin{aligned}
\rho_{ck}(\theta) = {} & \frac{N+1}{4\pi\alpha N(1+p^2)} \Big\{ e^{-2\pi\alpha\theta} \cos 2\pi\alpha p\theta \times \\
& \times \Big[(3-p^2)(1 - \mathrm{ch}\, 2\pi\alpha \cos 2\pi\alpha p) + \frac{1-3p^2}{p} \times \\
& \times \mathrm{sh}\, 2\pi\alpha \sin 2\pi\alpha p\Big] + e^{-2\pi\alpha\theta} \sin 2\pi\alpha p\theta \times \\
& \times \Big[(p^2-3)\, \mathrm{sh}\, 2\pi\alpha \sin 2\pi\alpha p + \frac{1-3p^2}{p} \times \\
& \times (1 - \mathrm{ch}\, 2\pi\alpha \cos 2\pi\alpha p)\Big]\Big\}. \qquad (7.21)
\end{aligned}
$$

Figures 7.2, 7.3, and 7.4 show the envelopes of the autocorrelation function of a phase-manipulated signal which has passed through a system of two identical coupled circuits under different values of the band coefficient α, coupling factor p, and sequence length N = 127. Analysis of these curves shows that:

1) for any values of α or p the envelope has a very rounded peak at the point of maximum correlation;

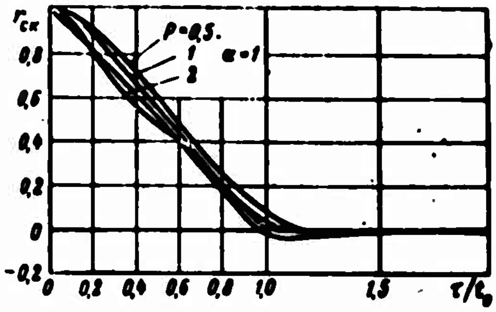

Fig. 7.2. Envelope of correlation function of phase-manipulated signal which has passed through a system of two coupled circuits.

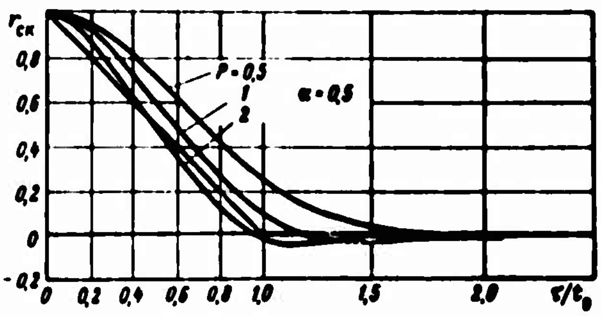

Fig. 7.3. Envelope of correlation function of phase-manipulated signal which has passed through a system of two coupled circuits.

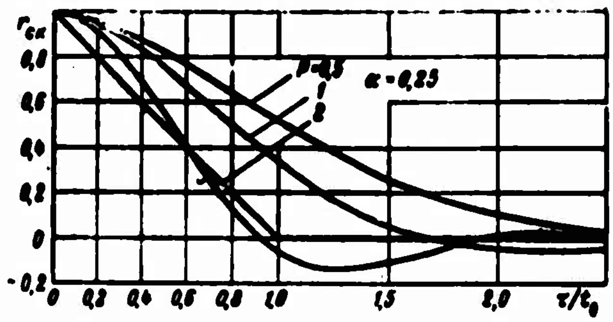

Fig. 7.4. Envelope of correlation function of phase-manipulated signal which has passed through a system of two coupled circuits.

2) for a coupling factor (p = 0.5) which is less than critical the envelope slowly declines with small oscillations toward an established value; the lower the passband of the filter (the lower the value of α), the greater will be the expansion in the main lobe of the envelope;

3) if coupling is greater than critical (p = 2), there is virtually no expansion in the main lobe, although thereafter there appear negative and positive spikes, which are comparable at low values of α to the value of the correlation function when θ = 0;

4) the case of critical coupling (p = 1) represents an intermediate between the two above.

3. Ideal Bandpass Filter

Let us examine the extreme case of filtering a phase-manipulated signal in which the amplitude-frequency characteristic is close to the characteristic of an ideal bandpass filter:

$$K(f)=\begin{cases} 1 & \text{при } |f-f_c|<\dfrac{B}{2}, \\ 0 & \text{при } |f-f_c|>\dfrac{B}{2}. \end{cases} \qquad (7.22)$$

[при = when]

As already demonstrated, the energy spectrum of a periodic pseudorandom sequence is discrete in view of the regular nature of the process. This fact creates some inconvenience in our study on the behavior of the correlation function with changes in the passband of the filter, since the spectrum samples in this case should theoretically be cut off by the characteristic of the ideal filter in a jump. Here it should be mentioned that this approximation of the frequency characteristic does not correspond to the one which is physically attainable. The steepness of the slopes of the frequency characteristic of an actual filter is always finite, while within the transmission band the filter will always have a nonzero transmission coefficient on the finite frequency section. Thus, it is desirable to use an approach in which the spectrum of the pseudorandom sequence will be unbroken and continuous, which corresponds to the nonperiodic correlation function.

Let us examine the correlation function of a periodic pseudorandom sequence in the form of the sum of the functions, i.e.,

$$r_{\text{п}}(\tau)=-\frac{1}{N}+\sum_{l=-\infty}^{\infty} r^{\text{нп}}(\tau-lNt_0), \qquad (7.23)$$

where

$$r^{\text{нп}}(\tau)=\begin{cases} \dfrac{N+1}{N}\left(1-\dfrac{|\tau|}{t_0}\right) & \text{при } 0<\dfrac{|\tau|}{\tau_0}<1, \\ 0 & \text{at other values of } \tau. \end{cases} \qquad (7.24)$$

The energy spectrum which corresponds to the correlation function of (7.23) will have the form of

$$S_{\text{и}}(f) = -\frac{S(f)}{N} + \sum_{l=-\infty}^{\infty} e^{-i2\pi f l N t_0} S^{\text{нп}}(f). \qquad (7.25)$$

where the spectrum $S^{\text{нп}}(f)$ corresponds to the correlation function of (7.24). In this case the envelope of the autocorrelation function of the phase-manipulated signal which has passed through the ideal filter is equal to

$$r_{\text{иф}}(\tau) = \frac{1}{k_{\text{н}}^{\text{иф}}} \int_{-\infty}^{\infty} S_{\text{и}}(f) K^2(f+f_c) e^{i2\pi f\tau} d\tau =$$

$$= -\frac{1}{Nk_{\text{н}}^{\text{иф}}} + \frac{1}{k_{\text{н}}^{\text{иф}}} \sum_{l=-\infty}^{\infty} \rho_{\text{иф}}(\tau - lNt_0). \qquad (7.26)$$

where $k_{\text{н}}^{\text{иф}}$ is the normalizing factor, and the function $\rho_{\text{иф}}(\tau)$ is equal to

$$\rho_{\text{иф}}(\tau) = \int_{-\infty}^{\infty} S^{\text{нп}}(f) K^2(f+f_c) e^{i2\pi f\tau} df =$$

$$= \int_{-\frac{B}{2}}^{+\frac{B}{2}} S^{\text{нп}}(f) e^{i2\pi f\tau} df =$$

$$= \int_{-\frac{B}{2}}^{+\frac{B}{2}} \int_{-t_0}^{+t_0} r^{\text{нп}}(x) \cos 2\pi f x \cos 2\pi f \tau \, df \, dx.$$

Integrating this expression involves no theoretical difficulty. After substituting expression (7.24) and introducing, as before, the dimensionless band coefficient $\alpha = (Bt_0)/2$, we get

$$\rho_{\text{иф}}(\theta) = \frac{N+1}{N\pi} \Big\{ \mathrm{Si}(2\pi\alpha + 2\pi\alpha\theta) + \mathrm{Si}(2\pi\alpha - 2\pi\alpha\theta) +$$

$$+ \theta [\mathrm{Si}(2\pi\alpha + 2\pi\alpha\theta) - \mathrm{Si}(2\pi\alpha - 2\pi\alpha\theta) - 2\mathrm{Si}(2\pi\alpha\theta)] -$$

$$- \frac{2}{\pi\alpha} \sin^2 \pi\alpha \cos 2\pi\alpha\theta \Big\}, \qquad (7.27)$$

where $\mathrm{Si}\,x = \int_0^x \frac{\sin y}{y}\,dy$ is the integral sine; $\theta = \tau_0/t_0$ is relative delay time.

For rather long sequences the envelope of the correlation function of the filtered phase-manipulated signal is practically determined by the zero term of the sum of (7.26) alone. For this case we have

$$r_{\text{нф}}(\theta) = -\frac{1}{N k_{\text{н}}^{\text{нф}}} + \frac{\rho_{\text{нф}}(\theta)}{k_{\text{н}}^{\text{нф}}}, \qquad (7.28)$$

where the normalizing coefficient $k_{\text{н}}^{\text{нф}}$ equals

$$k_{\text{н}}^{\text{нф}} = -\frac{1}{N} + \rho_{\text{нф}}(0) = -\frac{1}{N} + \frac{2(N+1)}{N\pi}\left[\mathrm{Si}(2\pi\alpha) - \frac{1}{\pi\alpha}\sin^2 \pi\alpha\right]. \qquad (7.29)$$

The envelope of the normalized autocorrelation function of a signal which is passed through the ideal filter is shown in Fig. 7.5 for a sequence length of N = 127. The distinguishing feature of these curves as compared to the two above is their slightly damped nature.

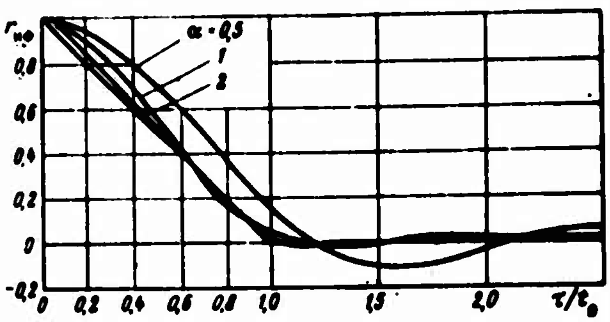

Fig. 7.5. Envelope of correlation function of phase-manipulated signal which has passed through an ideal filter.

7.3. CHARACTERISTICS OF ENVELOPE OF CORRELATION FUNCTION DURING SIGNAL FILTRATION

In studying the envelope of the correlation function of a signal we are also interested in such parameters as the width of the main lobe of the correlation function, the position and level of the side lobes, and correlation time. Let us examine the nature of change in these parameters during filtration of the signal. The high-correlation range can easily be characterized by a wide correlation function, which we will define as a time segment calculated from the value of the envelope maximum up to the moment in time corresponding to a level of 0.5 (-3 dB) from the maximal.

In order to obtain a good estimate of potential discrimination we will determine correlation time. In many cases correlation time is understood as the time segment calculated from the maximal value of the correlation function up to the moment and time at which the value of the correlation function is less than a preassigned value (for example, 0.1, 0.01, etc.). This determination of the correlation time is most frequently used for signals whose correlation function asymptotically approaches zero. The correlation function of a phase-manipulated signal does not have this property. Characteristic of this signal is the presence of residual, side lobes, whose magnitude depends on the length of the sequence. It has been shown that the envelope of the autocorrelation function of a phase-manipulated signal falls toward an established value when delay θ increases. Analysis of the formulas of (7.12), (7.17), and (7.28) shows that this established value is equal to $-1/Nk_H$, where k_H is the normalizing coefficient for each type of filter, respectively. Thus it makes sense not to stipulate in advance the reference level of the correlation function in determining correlation time but to consider it dependent on the established value. Let us determine correlation time as a time interval between the maximal value of the correlation function and the moment corresponding to the condition that the correlation

function lies between zero and twice the value of the established quantity. In determining the magnitude of negative and positive spikes we will find their absolute value and location on the time axis by counting from the maximum point of the correlation function.

1. Width of Main Lobe

In order to find the width of the envelope of the autocorrelation function of a filtered phase-manipulated signal we must find the root of the equation

$$r_{\Phi}(\theta_{ш}) = +0{,}5,$$

where $\theta_{ш}$ is the unknown width; $r_{\Phi}(\theta)$ is the analytical expression of the envelope, which for the studied types of filters is determined by (7.12), (7.17), and (7.28).

Analysis shows that the obtained equations are transcendental. The results of the graphic solution are shown in Fig. 7.6. Curves expressed as a broken line are asymptotic curves, to which the unknown values of $\theta_{ш}$ strive at low values of α. Each of these curves describes half of the width of the correlation function at a level of -3 dB for the random process obtained when white noise is passed through the studied linear filter.

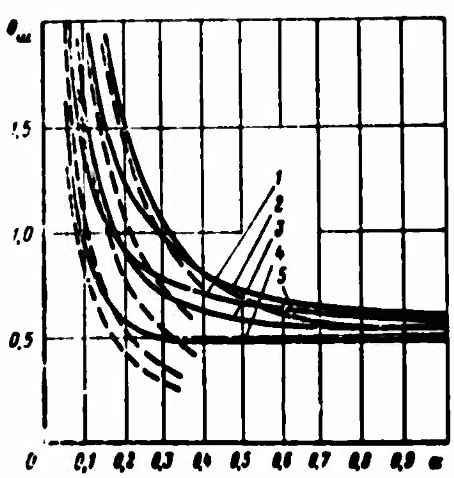

Fig. 7.6. Change in width of correlation function envelope during filtration: 1 - single oscillatory circuit; 2, 3, 4 - system of two coupled circuits for coupling factors of 0.5, 1, and 2; 5 - ideal filter.

From these graphics representing dependences we find that for the studied types of filters the widening in the correlation function when $\alpha \geq 0.6$ is not great. For small passband values, when $\alpha \leq 0.4$, the widening is substantial, and the curves rise steeply.

2. Correlation Time

From determining correlation time we find that this is the greatest value of the roots of the two equations

$$\left.\begin{aligned} r_{\Phi}(\theta_{к}) &= 0, \\ r_{\Phi}(\theta_{к}) &= -\frac{2}{N k_{и}} \end{aligned}\right\}. \qquad (7.30)$$

The solution of these equations based on (7.12) and (7.15) for the envelope of the autocorrelation function of a phase-manipulated signal which has passed a single oscillatory circuit provides the following quantity for correlation time:

$$\theta_{к} = \frac{1}{2\pi\alpha} \ln \frac{(N+1)\,\mathrm{sh}^2\,\pi\alpha}{\pi\alpha} \qquad (7.31)$$

For a bandpass filter the equations of (7.30) are reduced on the basis of expression (7.17) to the following:

$$\left.\begin{aligned} \rho_{ск}(\theta_{к}) &= \frac{1}{N}, \\ \rho_{ск}(\theta_{к}) &= -\frac{1}{N}. \end{aligned}\right\} \qquad (7.32)$$

where $\rho_{CK}(\theta)$ is determined by realtionship (7.21). Finding correlation time indirectly from these formulas leads to the solution of an awkward transcendental equation, whose approximate root value is unknown. Let us estimate correlation time from the second-order envelope (envelope of the autocorrelation function envelope of a filtered pseudorandom signal). We will represent function $\rho_{CK}(\theta)$ [expression (7.21)] in the form of the product of the exponential function and the cosinusoidal

$$\rho_{c\kappa}(\theta) = Ae^{-2\pi\alpha\theta}\cos(2\pi\alpha p\theta - \varepsilon), \qquad (7.33)$$

where

$$A = \frac{(N+1)\sqrt{1+p^2}}{4\pi\alpha N p} \times$$

$$\times \sqrt{(1 - \operatorname{ch} 2\pi\alpha \cos 2\pi\alpha p)^2 + \operatorname{sh}^2 2\pi\alpha \sin^2 2\pi\alpha p},$$

$$\varepsilon = \operatorname{arctg} \frac{-(3-p^2)\operatorname{sh} 2\pi\alpha \sin 2\pi\alpha p + \frac{1-3p^2}{p}(1 - \operatorname{ch} 2\pi\alpha \cos 2\pi\alpha p)}{(3-p^2)(1 - \operatorname{ch} 2\pi\alpha \cos 2\pi\alpha p) + \frac{1-3p^2}{p}\operatorname{sh} 2\pi\alpha \sin 2\pi\alpha p}.$$

Then the equation for finding correlation time from the second-order envelope will have the form of

$$Ae^{-2\pi\alpha\theta_\kappa} = \frac{1}{N}.$$

Hence the correlation time of a signal which has passed through a system consisting of two coupled circuits is determined by the relationship

$$\theta_\kappa = \frac{1}{2\pi\alpha} \ln \frac{(N+1)\sqrt{1+p^2}\sqrt{(1 - \operatorname{ch} 2\pi\alpha \cos 2\pi\alpha p)^2 + \operatorname{sh}^2 2\pi\alpha \sin^2 2\pi\alpha p}}{4\pi\alpha p}. \qquad (7.34)$$

It has been demonstrated that the envelope of the correlation function of a phase-manipulated system which has passed an ideal bandpass filter has an oscillating, slightly damped nature. In this case correlation time is determined by the same method as used in the case of a bandpass filter, i.e., from the second-order envelope. The necessary calculation work with respect to (7.27), (7.28), and (7.29) is done on the electronic computer. The results of processing these data and the correlation time values for the two pseudorandom sequences N = 127 and 1023, calculated according to the formulas of (7.31) and (7.34), are shown graphically in Fig. 7.7. In Fig. 7.7 the curves corresponding to a bandpass filter at coupling factors p = 0.5 and 1 are practically between curves 1 and 2 and are therefore not expressed.

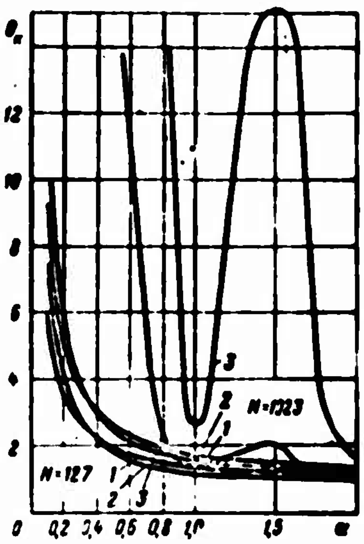

Fig. 7.7. Change in correlation time of phase-manipulated signals during filtration: 1 - single oscillatory circuit; 2 - bandpass filter at coupling factor of p = 2; 3 - ideal bandpass filter.

Comparison of these correlation time dependences leads to the conclusion that a linear filter which is close to ideal, i.e., which has very steep slopes and extremely small transmission coefficient values within the passband, can substantially impair the correlation properties of the filtered phase-manipulated signal.

3. Magnitude and Position of Negative and Positive Spikes

Our definition of spikes [peaks] in the envelope of the correlation function would be all extreme points with the exception of the maximum point at zero delay. The coordinate of the extreme point is the root of equaiton $r'_{\Phi}(\theta) = 0$. The solution to this equation for the envelope of aphase-manipulated signal which has passed through the oscillatory circuit gives us the only root value $\theta = 0$. Thus, in this case the envelope does not have spikes. As follows from Figs. 7.2, 7.3, and 7.4, the spikes in the envelope of the correlation function of a phase-manipulated signal which is passed through a system of two coupled circuits begin after $\theta = 1$. It is obvious in this case that finding the extreme points based on (7.17) involves solving the equation

$$r'_{cx}(\theta)=\left[\frac{\frac{1}{N}-\rho_{cx}(\theta)}{k_{M}^{\kappa}}\right]'=\rho'_{cx}(\theta)=0.$$

In finding the roots of the equation we substitute the value of the function in the form of (7.33). Having taken the derivative and solved the equation, we get the following values for the x-coordinates of the extreme points:

$$\theta_{м} = \frac{\varepsilon - \operatorname{arctg}\frac{1}{p} + \pi n}{2\pi\alpha p}, \text{ где } n = 0, 1, 2, \ldots$$

[где = where]

If we substitute this quantity in (7.17), then we can show that the position of the k-th maximum and the k-th minimum of the envelope and the value of the envelope at the corresponding extreme point are determined by the relationships

$$\theta_{k_{min}} = \theta_{1м} + \frac{k-1}{\alpha p},$$

$$r_{сн}(\theta_{k_{min}}) = \frac{1}{\sqrt{k_{н}^{сн}}} - \frac{2\rho_{сн}(\theta_{1м})}{k_{н}^{сн}} e^{-\frac{2\pi(k-1)}{p}},$$

$$\theta_{k_{max}} = \theta_{1м} + \frac{2k-1}{2\alpha p},$$

$$r_{сн}(\theta_{k_{max}}) = \frac{1}{\sqrt{k_{н}^{сн}}} + \frac{2\rho_{сн}(\theta_{1м})}{k_{н}^{сн}} e^{-\frac{\pi(2k-1)}{p}},$$

where k = 1, 2, 3, ...;

$$\theta_{1м} = \frac{1}{2\pi\alpha p}\left(\varepsilon - \operatorname{arctg}\frac{1}{p}\right);$$

$$\rho_{сн}(\theta_{1м}) = \frac{N+1}{4\pi\alpha N} \times$$

$$\times \sqrt{(1 - \operatorname{ch}2\pi\alpha \cos 2\pi\alpha p)^2 + \operatorname{sh}^2 2\pi\alpha \sin^2 2\pi\alpha p}\; e^{-\frac{\varepsilon - \operatorname{arctg}\frac{1}{p}}{p}}.$$

In order to simplify analysis the series of dependences in the formulas found above is represented in the form of curves. The value of the normalizing coefficient k_{H}^{CH} and the position of the first minimum of the envelope θ_{1M} as a function of the band coefficient α for three coupling factors p are represented in Figs. 7.8 and 7.9. The values of the quantity $\rho_{CH}(\theta_{1M})/k_{H}^{CH}$, which

was calculated under the assumption that for sufficiently long sequences $N + 1/N \approx 1$, are shown in Figs. 7.10, 7.11, and 7.12.

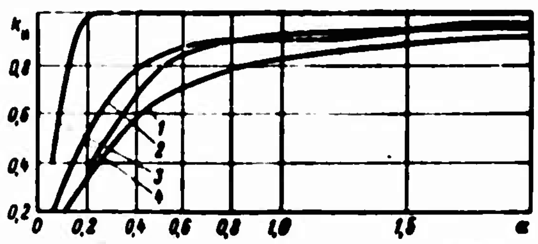

Fig. 7.8. Normalizing coefficient: 1, 2, 3 - bandpass filter for coupling factors of p = 0.5, 1, 2; 4 - ideal filter.

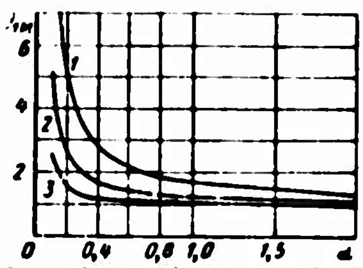

Fig. 7.9. Coordinate of first envelope minimum: 1, 2, 3 - bandpass filter for coupling factors of p = 0.5, 1, 2.

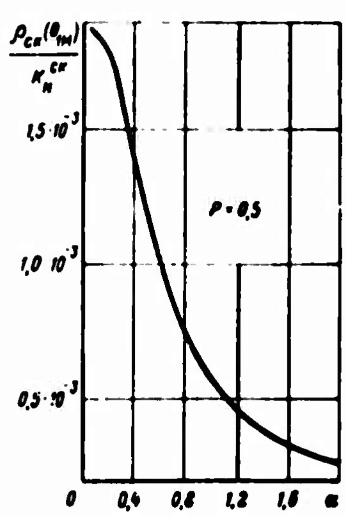

Fig. 7.10. Curve of normalizing function of effect.

The final relationships for the envelope of the correlation function of a phase-manipulated signal which has passed an ideal filter are expressed as integral sines, and thus finding the coordinates of the spikes involves solving complex transcendental equations. The coordinates of the spikes are found on the basis of analyzing the values of the envelope for certain quantities of the band coefficient α (calculations performed by means of a digital electronic computer). The obtained values for the coordinate of the first spike $\theta_{1м}$ (minimum) and the second spike $\theta_{2м}$ (maximum) of the envelope are shown in Fig. 7.13. The magnitude of envelopes at these points according to (7.28) will be

$$r_{иф}(\theta_{1м}) = -\frac{1}{Nk_н^{иф}} - \frac{\rho_{иф}(\theta_{1м})}{k_н^{иф}},$$

$$r_{иф}(\theta_{2м}) = -\frac{1}{Nk_н^{иф}} \quad \frac{\rho_{иф}(\theta_{2м})}{k_н^{иф}}.$$

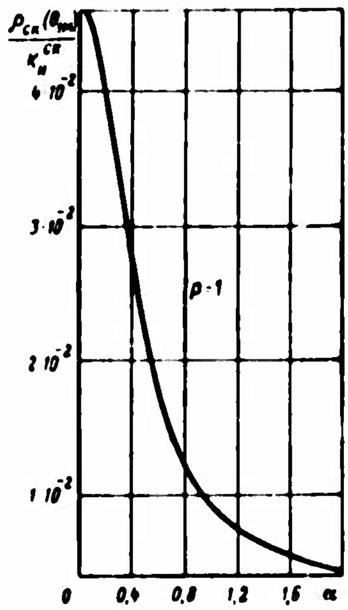

Fig. 7.11. Curve of normalizing function of effect.

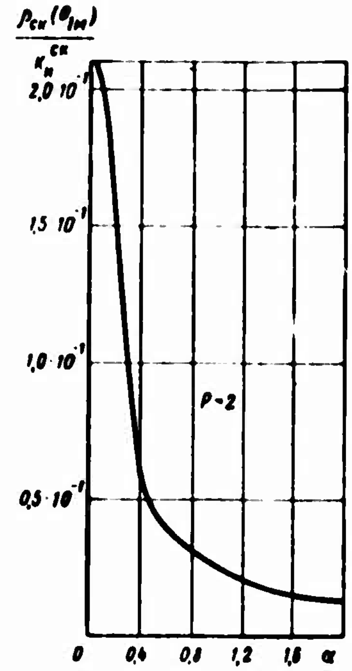

Fig. 7.12. Curve of normalizing function of effect.

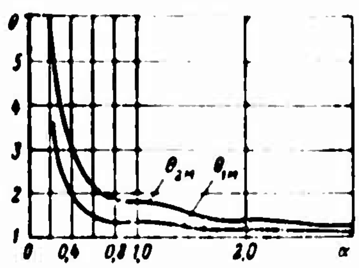

Fig. 7.13. [illegible] phase-manipulated [illegible]

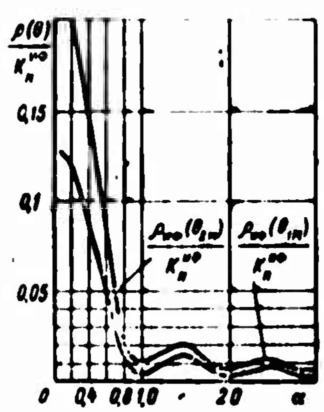

Fig. 7.14. Curve of normalized effect function.

The values $\frac{\rho_{нф}(\theta_{1m})}{k_н^{нф}}$, $\frac{\rho_{нф}(\theta_{2m})}{k_н^{нф}}$ are plotted as a function of the band coefficient α in Fig. 7.14. The normalizing coefficient $k_н^{нф}$ is found from the curves of Fig. 7.8.

7.4. EFFECT OF THE SHAPE OF AMPLITUDE-FREQUENCY CHARACTERISTIC OF FILTER ON CORRELATION FUNCTION OF FILTERED SIGNAL

In the preceding sections we have looked at the properties of the correlation function of a phase-manipulated signal which has passed through filters of different types

with different characteristics. It is interesting to examine the effect of the shape of the frequency characteristic of the filter on the correlation function of a filtered signal when the passband on a level of -3 dB is the same for all types of filters. Let us plot the envelopes of the correlation functions for one particular case where the passband of the filtering system is equal to the width of the energy spectrum of the pseudorandom sequence with respect to the first zeroes of the envelope, i.e., $\Delta f = 2f_{\text{такт}}$. Figure 7.15 shows diagrams of the amplitude-frequency characteristics of several filters which have identical passbands. In the same figure curve 6 represents the envelope of the energy spectrum of a pseudorandom sequence. The corresponding envelopes of the correlation functions of phase-manipulated signals calculated from formulas obtained in this chapter are shown in Fig. 7.16.

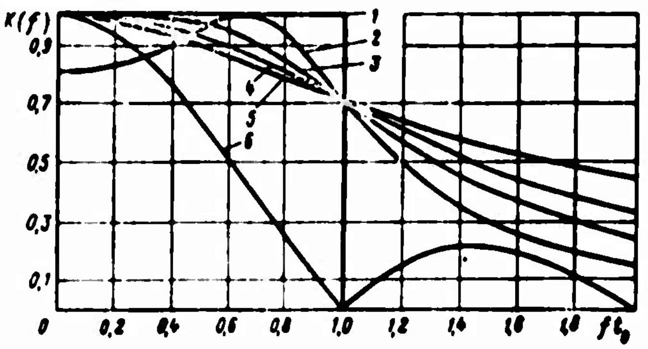

Fig. 7.15. Frequency characteristics of different filters with the same passbands: 1 - ideal bandpass filter $\alpha = 1$; 2 - coupled circuits at $p = 2$, $\alpha \approx 0.4$; 3 - coupled circuits for critical coupling $p = 1$, $\alpha = 0.7$; 4 - coupled circuits with weak coupling $p = 0.5$, $\alpha = 1.2$; 5 - single oscillatory circuit $\alpha = 1$.

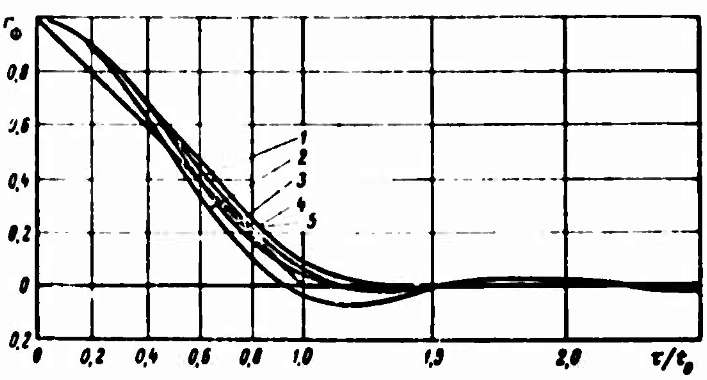

Fig. 7.16. Envelopes of autocorrelation functions of phase-manipulated signal during filtration: 1 - bandpass filter $\alpha = 1$; 2 - coupled circuits at $p = 2$, $\alpha = 0.4$; 3 - coupled circuits at critical coupling $p = 1$, $\alpha = 0.7$; 4 - coupled circuits for weak connection $p = 0.5$, $\alpha = 1.2$; 5 - single oscillatory circuit $\alpha = 1$.

Analysis of the diagrams leads to the conclusion that curves 1, 3, 4, and 5 differ little from the autocorrelation function of an undistorted pseudorandom sequence. This cannot be said of curve 2, which was plotted for a system of coupled circuits with a strong connection of p = 2. In the last case the amplitude-frequency characteristic of the filter has a two-humped nature with a dip at the maximum point of the energy spectrum of the signal. The envelope of the correlation function of a filtered phase-manipulated signal will differ little from the autocorrelation function of an undistorted pseudorandom sequence in the high-correlation range if the amplitude-frequency characteristic of the linear filter causes the least deformation in the main lobe of the energy spectrum of the signal.

EIGHTH CHAPTER

EFFECT OF LINEAR CHANNELS ON CORRELATION PROCESSING OF PHASE-MANIPULATED SIGNALS

8.1. ANALYSIS OF LINEAR TRANSFORMATIONS IN CORRELATED SYSTEM

If we use correlation processing of phase-manipulated signals, then we must study the effect of the parameters of linear filters in correlator channels on the measured cross-correlation function. It is obvious that the absence of agreement of frequency characteristics of filters with signal spectra will lead to systematic error in measuring signal delay due to the difference in group delays of both channels. Let us look at some features of processing phase-manipulated signals in the system whose block diagram is shown in Figs. 3.3 and 3.5.

The expressions for the normalized cross-correlation function of phase-manipulated signals at the output of filters Φ_1 and Φ_2 are found by the same method as relationship (6.3). As a result we get

$$\left.\begin{aligned} r_{12}(\tau) &= \\ r_{21}(\tau) &= \end{aligned}\right\} \mathrm{Re}\,\frac{1}{\sigma_1\sigma_2}\int_{-\infty}^{\infty} G_0(f)\,K_1(if)\,K^*_2(if)\begin{cases} e^{-i2\pi f\tau} \\ e^{i2\pi f\tau} \end{cases} df. \qquad (8.1)$$

where $G_c(f)$ is the energy spectrum of the input signal; σ_1, σ_2 are effective values, equal to

$$\sigma_p = \sqrt{\int_{-\infty}^{\infty} G_c(f)\, K_p^2(f)\, df}, \quad p = 1;\ 2.$$

The energy spectrum of a phase-manipulated signal based on (7.25) and (7.[illegible]) for the positive frequency range will be written in the form of

$$G_c(f) = \frac{1}{N}\,\delta(f - f_c) + \sum_{l=-\infty}^{\infty} e^{-i2\pi(f - f_c)lNt_0}\, S^{nn}(f - f_c). \qquad (8.2)$$

The energy spectrum $S^{nn}(f)$ corresponding to the correlation function of (7.24) can be found by means of the direct Wiener-Khinchin transform.

$$S^{nn}(f) = \frac{(N+1)\,t_0\,(1 - \cos 2\pi f t_0)}{2N(\pi f t_0)^2}. \qquad (8.3)$$

In the relationships of (8.1) we substitute (8.2) and by making the replacement $x = f - f_c$ in the integrals, we get

$$\left.\begin{array}{l} r_{12}(\tau) = \\ r_{21}(\tau) = \end{array}\right\} \frac{1}{\sigma_1\sigma_2}\,\mathrm{Re}\left\{\begin{array}{l} e^{-i2\pi f_c\tau} \\ e^{i2\pi f_c\tau} \end{array}\right\} \times$$

$$\times\left[\frac{1}{N}K_1(if_c)\,K_2^*(if_c) + \sum_{l=-\infty}^{\infty}\left\{\begin{array}{l} B_{12}^{nn}(\tau - lNt_0) \\ B_{21}^{nn}(\tau - lNt_0) \end{array}\right\}\right], \qquad (8.4)$$

where we have

$$\left.\begin{array}{l} B_{12}^{nn}(\tau) = \\ B_{21}^{nn}(\tau) = \end{array}\right\} \int_{-\infty}^{\infty} S^{nn}(f)\, K_1[i(f + f_c)] \times$$

$$\times K_2^*[i(f + f_c)]\left\{\begin{array}{l} e^{-i2\pi f\tau} \\ e^{i2\pi f\tau} \end{array}\right\} df. \qquad (8.5)$$

We will assume that the length of the sequence is sufficient and the passbands of linear filters Φ_1 and Φ_2 have the same width as the band of the energy spectrum of the signals. With a greater degree of accuracy we can assume that the unknown cross-correlation function in the range of $|\tau| \leqslant \frac{1}{2} N t_0$ can be practically determined by the zero term of the sum of (8.4) and, consequently,

$$\left.\begin{matrix} r_{12}(\tau) = \\ r_{21}(\tau) = \end{matrix}\right\} \frac{1}{\sigma_1\sigma_2} \operatorname{Re}\left\{\begin{matrix} e^{-i2\pi f_c\tau} \\ e^{i2\pi f_c\tau} \end{matrix}\right\} \times$$
$$\times\left[\left\{\begin{matrix} B_{12}^{\text{нп}}(\tau) \\ B_{21}^{\text{нп}}(\tau) \end{matrix}\right\} - \frac{K_1(if_c)K_2^*(if_c)}{N}\right]. \qquad (8.6)$$

We substitute (5.3) in (8.5). By making a replacement with variable $f = x/t_0$ and designating delay time $\theta = \tau/t_0$, we get

$$\left.\begin{matrix} B_{12}^{\text{нп}}(\theta) = \\ B_{21}^{\text{нп}}(\theta) = \end{matrix}\right\} \frac{N+1}{2N\pi^2} \int_{-\infty}^{\infty} \frac{1}{f^2} K_1\left[i\left(\frac{f}{t_0}+f_c\right)\right] \times$$
$$\times K_2^*\left[i\left(\frac{f}{t_0}+f_c\right)\right] \times$$
$$\times\left\{\begin{matrix} e^{-i2\pi f\theta} - 0{,}5e^{-i2\pi f(\theta-1)} - 0{,}5e^{-i2\pi f(\theta+1)} \\ e^{i2\pi f\theta} - 0{,}5e^{i2\pi f(\theta-1)} - 0{,}5e^{i2\pi f(\theta+1)} \end{matrix}\right\} df. \qquad (8.7)$$

For the sake of abbreviation we introduce the following symbols for the integrals

$$\left.\begin{matrix} I_1(\theta) = \\ I_2(\theta) = \end{matrix}\right\} \frac{N+1}{2N\pi^2} \int_{-\infty}^{\infty} \frac{1}{f^2} K_1\left[i\left(\frac{f}{t_0}+f_c\right)\right] \times$$
$$\times K_2^*\left[i\left(\frac{f}{t_0}+f_c\right)\right] \left\{\begin{matrix} e^{-i2\pi f\theta} \\ e^{i2\pi f\theta} \end{matrix}\right\} df. \qquad (8.8)$$

Let us limit ourselves to examining the normalized cross-correlation function for a case where the linear filters are n-stage resonance amplifiers, each stage of which has a transmission coefficient, as in the single-circuit amplifier (see § 6.2).

In analyzing it is desirable to use certain generalized coefficients rather than the absolute values of the parameters of the filters and the characteristics of the phase-manipulated signal:

$\alpha_1 = 0.5B_1 t_0$; $\alpha_2 = 0.5B_2 t_0$ - the bandpass coefficients;

$\beta_1 = \frac{2(f_0 - f_1)}{B_1}$; $\beta_2 = \frac{2(f_0 - f_2)}{B_2}$ - the coefficients of shifts in resonance frequencies.

In the new symbols the integrals of (8.8), taking into account the expressions for the transmission coefficients of the resonance amplifiers, will have the form of

$$\left.\begin{aligned} I_1(\theta) &= \\ I_2(\theta) &= \end{aligned}\right\} \frac{(N+1)(\alpha_1\alpha_2)^n}{2N\pi^2} \int\limits_{-\infty}^{\infty} \left\{\begin{matrix} e^{-i2\pi f\theta} \\ e^{i2\pi f\theta} \end{matrix}\right\} \times$$
$$\times [f - (-\beta_1\alpha_1 + i\alpha_1)]^{-n} [f + (\beta_2\alpha_2 + i\alpha_2)]^{-n} \frac{df}{f^2}.$$

These integrals are calculated by means of the remainder theorem. For integral $I_1(\theta)$ the selected integration contour consists of the actual axis of frequencies f and the semicircle of infinite radius in the lower half-plane. Then, according to the Jordan lemma and the Cauchy theorem, the unknown integral is determined as the sum of remainders relative to poles located beneath the material axis. Note that the integrand function has a pole of the second order on the material axis itself. In this case according to [1, p. 64] one should add to the sum of the remainders for poles located under the actual axis half of the sum of the remainders for poles on the material axis itself. By using the formulas for calculating the remainders for multiple poles, we get

$$I_1(\theta) = \frac{(N+1)(\alpha_1\alpha_2)^n}{2N\pi^2} \left\{(-\pi i)\frac{d}{df} \times \right.$$
$$\times \left[\frac{e^{-i2\pi f\theta}}{(f + \beta_1\alpha_1 - i\alpha_1)^n (f + \beta_2\alpha_2 + i\alpha_2)^n}\right]_{f=0} +$$
$$\left. + (-2\pi i)\frac{1}{(n-1)!} \cdot \frac{d^{n-1}}{df^{n-1}} \cdot \left[\frac{e^{-i2\pi f\theta}}{f^2 (f + \beta_1\alpha_1 - i\alpha_1)^n}\right]_{f=-\beta_2\alpha_2 - i\alpha_2}\right\}. \quad (8.9)$$

In calculating integral $I_2(\theta)$ the integration contour is the material axis, closed by a semicircle of infinite radius, which is located in the upper half-plane.

If we use the Leibniz rule for calculating the derivative of the $(n - 1)$-order from the product of the two functions, then after rather laborious nonfundamental calculations we get the following expressions for the unknown integrals:

$$
\begin{aligned}
I_1(\theta) = &-\frac{(N+1)\theta}{N(1+i\beta_1)^n(1-i\beta_2)^n} - \\
&-\frac{(N+1)n[(\alpha_2-\alpha_1)-i(\beta_2\alpha_2+\beta_1\alpha_1)]}{2N\pi\alpha_1\alpha_2(1+i\beta_1)^{n+1}(1-i\beta_2)^{n+1}} - \frac{(N+1)(\alpha_1\alpha_2)^n}{N\pi}\times \\
&\times e^{-2\pi\alpha_2\theta(1-i\beta_2)}\sum_{l=0}^{n-1}\sum_{k=0}^{n-l-1}\frac{(n+l-1)!(k+1)}{l!(n-1)!(n-k-l-1)!}\times \\
&\times[(\alpha_2+\alpha_1)-i(\beta_2\alpha_2-\beta_1\alpha_1)]^{-n-l}(\alpha_2-i\beta_2\alpha_2)^{-2-k}\times \\
&\times(2\pi\theta)^{n-l-k-1}.
\end{aligned}
\tag{8.10}
$$

$$
\begin{aligned}
I_2(\theta) = &-\frac{(N+1)\theta}{N(1+i\beta_1)^n(1-i\beta_2)^n} + \\
&+\frac{(N+1)n[(\alpha_2-\alpha_1)-i(\beta_2\alpha_2+\beta_1\alpha_1)]}{2N\pi\alpha_1\alpha_2(1+i\beta_1)^{n+1}(1-i\beta_2)^{n+1}} - \frac{(N+1)(\alpha_1\alpha_2)^n}{N\pi}\times \\
&\times e^{-2\pi\alpha_1\theta(1+i\beta_1)}\sum_{l=0}^{n-1}\sum_{k=0}^{n-l-1}\frac{(n+l-1)!(k+1)}{l!(n-1)!(n-l-k-1)!}\times \\
&\times[(\alpha_2+\alpha_1)-i(\beta_2\alpha_2-\beta_1\alpha_1)]^{-n-l}(\alpha_1+i\beta_1\alpha_1)^{-2-k}\times \\
&\times(2\pi\theta)^{n-l-k-1}.
\end{aligned}
\tag{8.11}
$$

The normalized cross-correlation function of the phase-manipulated signals at the output of linear filters based on (8.6), (8.7), and (8.8) will have the form of

a) $0 \leq \theta \leq 1$

$$
\left.\begin{aligned} r_{12}(\theta) = \\ r_{21}(\theta) = \end{aligned}\right\} \frac{1}{\sigma_1\sigma_2}\,\mathrm{Re}\begin{Bmatrix} e^{-i2\pi f_0 t_0\theta} \\ e^{i2\pi f_0 t_0\theta} \end{Bmatrix}\Big[-\frac{1}{N}(1+i\beta_1)^{-n}(1-i\beta_2)^{-n} + \\
+\begin{Bmatrix} I_1(\theta) - 0.5I_2(1-\theta) - 0.5I_1(1+\theta) \\ I_2(\theta) - 0.5I_1(1-\theta) - 0.5I_2(1+\theta) \end{Bmatrix}\Big].
\tag{8.12}
$$

b) $1 \leq \theta \leq N/2$

$$\left.\begin{matrix} r_{12}(\theta) = \\ r_{21}(\theta) = \end{matrix}\right\} \frac{1}{\sigma_1 \sigma_2} \cdot \mathrm{Re} \left\{\begin{matrix} e^{-i2\pi f_c \tau_0 \theta} \\ e^{i2\pi f_c \tau_0 \theta} \end{matrix}\right\} \left[\frac{-1}{N} (1+i\beta_1)^{-n} (1-i\beta_2)^{-n} + \right.$$

$$\left. + \left\{\begin{matrix} I_1(\theta) - 0{,}5 I_1(\theta-1) - 0{,}5 I_1(\theta+1) \\ I_2(\theta) - 0{,}5 I_2(\theta-1) - 0{,}5 I_2(\theta+1) \end{matrix}\right\} \right].$$

Effective values of σ_1 and σ_2 are found from (8.12) at particular values of parameters θ; α_1; α_2; β_1; β_2, i.e.,

$$\sigma_1 = \sqrt{-\frac{1}{N}(1+i\beta_1)^{-n}(1-i\beta_2)^{-n} + I_1(0) - 0{,}5 I_1(1-0) - 0{,}5 I_1(0+1)}$$

when $\theta = 0$; $\alpha_1 = \alpha_2 = \alpha_1$; $\beta_1 = \beta_2 = \beta_1$.

$$\sigma_2 = \sqrt{-\frac{1}{N}(1+i\beta_1)^{-n}(1-i\beta_2)^{-n} + I_2(0) - 0{,}5 I_1(1-0) - 0{,}5 I_2(0+1)}$$

when $\theta = 0$; $\alpha_2 = \alpha_1 = \alpha_2$; $\beta_2 = \beta_1 = \beta_2$. (8.13)

In order to verify the calculations it is interesting to compare the obtained results with the results of the preceding chapter. From (8.10)-(8.13) when $\alpha_1 = \alpha_2 = \alpha$; $\beta_1 = \beta_2 = 0$; $n = 1$ we can obtain the expression for the autocorrelation function of a phase-manipulated signal which has passed a single oscillatory circuit whose central frequency corresponds to the maximum of the energy spectrum of the signal. After substituting the indicated quantities, we get:

a) $0 \leq \theta \leq 1$

$$r_{0\kappa}(\theta) = \frac{\cos 2\pi f_c \tau}{\sigma^2} \left[1 - \frac{N+1}{N}\theta + \right.$$

$$\left. + \frac{N+1}{2\pi\alpha N} (e^{-2\pi\alpha} \operatorname{ch} 2\pi\alpha\theta - e^{-2\pi\alpha\theta}) \right].$$

b) $1 \leq \theta \leq N/2$

$$r_{0\kappa}(\theta) = \frac{\cos 2\pi f_c \tau}{\sigma^2} \left[-\frac{1}{N} + \frac{N+1}{2\pi\alpha N} e^{-2\pi\alpha\theta} (\operatorname{ch} 2\pi\alpha - 1) \right].$$

where

$$\sigma^2 = 1 - \frac{A+B}{2\pi\alpha N}\left(1 - e^{-2\pi\alpha}\right).$$

From (7.5) and (7.12) we get analogous expressions for the autocorrelation function if we use the approximate expressions of the effect function of (7.14) and (7.15). As already mentioned, these approximate expressions result in error not exceeding 0.005% for rather long sequences, where $\alpha N\pi \geq 5$. Consequently, the expressions for the cross-correlation function found in this chapter are extremely close to the original values, which can theoretically be obtained by synthesizing the series, as done in the preceding chapter.

The obtained analytical expressions for the cross-correlation function depend on argument θ and fixed parameters: N - the length of the sequence; n - the number of resonance stages in the filter; α_1, α_2 - the passband coefficients; β_1, β_2 - the coefficients of shift in the resonance frequencies. Since the passbands of linear filters Φ_1 and Φ_2 are linked to the passbands of a separate stage by relationships $B^{(n)}_1 = \varepsilon_n B_1$, $B^{(n)}_2 = \varepsilon_n B_2$, then it also makes sense to determine the complete coefficients of the passbands and the shift in resonance frequencies as follows:

$$\alpha_1^{(n)} = 0.5B_1^{(n)}t_0 = \varepsilon_n\alpha_1;\quad \alpha_2^{(n)} = 0.5B_2^{(n)}t_0 = \varepsilon_n\alpha_2.$$

$$\beta_1^{(n)} = \frac{2(f_c - f_1)}{B_1^{(n)}} = \frac{\beta_1}{\varepsilon_n};\quad \beta_2^{(n)} = \frac{2(f_c - f_2)}{B_2^{(n)}} = \frac{\beta_2}{\varepsilon_n},$$

where $\varepsilon_n = \sqrt{\sqrt[n]{2} - 1}$.

The complete coefficients for the passbands and shift in resonance frequencies provide, in addition to purely practical convenience, a way of better estimating the effect of filter parameters on the cross-correlation properties of filtered phase-manipulated signals, as will be shown below. The normalized

cross-correlation function was calculated by means of an electronic computer using relationships (8.10)-(8.13).

8.2. CORRELATION PROPERTIES OF PHASE-MANIPULATED SIGNAL WHICH HAS PASSED THROUGH AN n-STAGE RESONANCE FILTER

The expression for an autocorrelation function of a phase-manipulated signal which has passed an n-stage filter is obtained as the particular case of the cross-correlation function of the output of linear filters at parameter values of $\alpha_1 = \alpha_2 = \alpha$, and $\beta_1 = \beta_2 = \beta$.

It is a good idea to examine separately the effect of the passband and the shift in resonance frequencies on the nature of the autocorrelation function and its parameters. It is interesting to study the envelope of the autocorrelation function, since in most technical applications information concerning the properties of the signal, which consists of "high-frequency" charging [filling], is not used. Thus, we will study the effect of different factors on the correlation properties of the envelope.

1. Change in Passband

We will assume that there is no shift in resonance frequency, i.e., $\beta^{(n)} = 0$. For this case Fig. 8.1 shows the envelopes of the autocorrelation function of a filtered phase-manipulated signal at a sequence length $N = 127$, $n = 1$ and $n = 4$ is the number of stages, and at several values of the complete passband coefficient $\alpha^{(n)}$. For comparison Fig. 8.1 shows the envelope of the correlation function of an undistorted figure (curve a). Analysis of the curve shows that the envelope of the correlation function of the filtered signal:

1) has a very round peak at the point of maximum correlation, which increases the dispersion of signal delay in the presence of interferring noise;

2) widens along axis θ - the narrower the passband of the linear filter (the smaller the band coefficient $\alpha^{(n)}$), the broader will be the envelope along axis θ;

3) falls monotonically toward an established value as relative delay θ increases;

4) changes only slightly at a different number of stages n, if, of course, the total passband of the resonance filter in this case remains constant (passband coefficient $\alpha^{(n)}$ does not change with a change in the number of stages n).

This last statement is valid at least for the studied cases of $\alpha^{(n)} \geq 0.25$; n = 1-4.

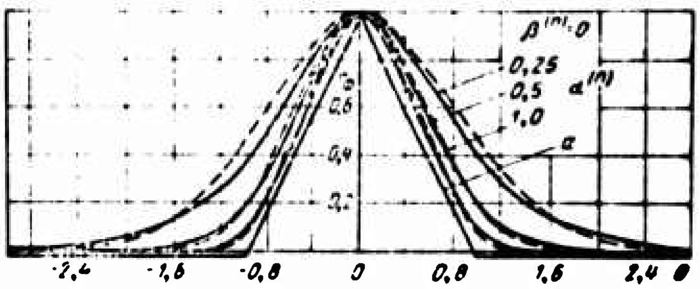

Fig. 8.1. Envelope of autocorrelation function of filtered phase-manipulated signal (—— n = 1; --- n = 4).

2. Resonance Frequency Shift

Figures 8.2 and 8.3 show the envelope of the autocorrelation function of a phase-manipulated signal at the output of a resonance amplifier for a sequence length of N = 127 and a stage number of n = 1 and 4. Comparison of envelopes corresponding to the presence

and absence of a resonance frequency shift ($\beta^{(n)} = 0; 1, 2$) leads to t e conclusion that the resonance frequency shift does not result in noticeable distortion of the envelope of the autocorrelation function of a filtered signal. The effect of the resonance frequency shift is noticed primarily in the form of a certain increase in correlation time, which even at high shift coefficient values of $\beta^{(n)} = 1.2$ is no more than 20% of the correlation time of an envelope when $\beta^{(n)} = 0$.

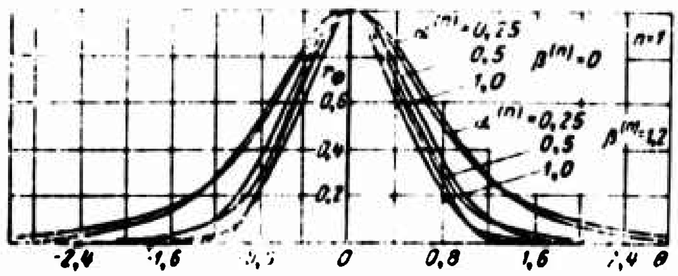

Fig. 8.2. Envelope of autocorrelation function of filtered phase-manipulated signal.

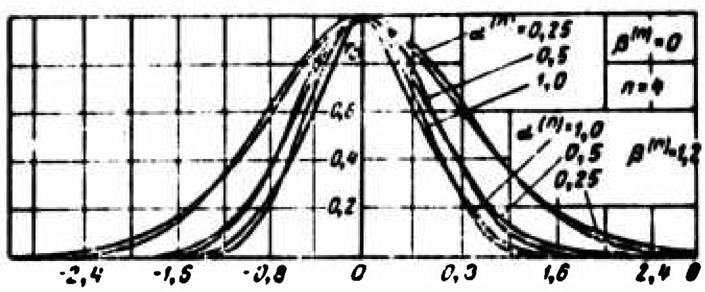

Fig. 8.3. Envelope of autocorrelation function of filtered phase-manipulated signal.

3. Accuracy Limit in Measuring Delay

The problems of the accuracy limit in measuring delay of a filtered phase-manipulated signal result from problems of reference ambiguity. Since a change in delay usually occurs from a maximal envelope value, then an essential requirement of reference ambiguity would be the absence (or negligibly small value of)

additional extreme values in the envelope near the point of maximal correlation. From Figs. 8.1-8.3, representing graphic dependences, and from additional plottings it follows that the envelope of the autocorrelation function of a phase-manipulated signal which has passed an n-stage resonance filter has when $|\theta| < N/2$ a single maximal value at any values of the band coefficient $\alpha^{(n)}$, shift in resonance frequency $\beta^{(n)}$, or number of cascades n, which is important from the standpoint of using filters to shape and process phase-manipulated signals.

Let us examine the parameters of a linear filter through which a signal passes on the potential (limiting) accuracy of measuring delay of a filtered phase-manipulated signal in the presence of noise. From the theory of estimating signal parameters [39] we know that in a case where the signal/noise ratio is rather high, dispersion in estimating the parameter is inversely proportional to the signal/noise ratio and the curvature of the normalized autocorrelation function of the useful signal for the estimated parameter at its maximum, i.e.,

$$\sigma_\lambda^2 = -\frac{1}{q^2 S''_\lambda(\lambda_0)},$$

where q^2 is the ratio of signal energy to the spectral density of the noise.

The accuracy limit in measuring delay, determined by dispersion in this delay, is expressed by the relationship

$$\sigma_\tau^2 = \frac{\tau_0^2}{q^2}\left[-\frac{1}{\rho''(\theta)}\right]\Big|_{\theta=0}.$$

The results of calculating the second derivative of the envelope of the autocorrelation function of a filtered signal when $\theta = 0$ as a function of the parameters of the linear filter n, $\alpha^{(n)}$, $\beta^{(n)}$ are shown in Fig. 8.4. From these dependences it follows that:

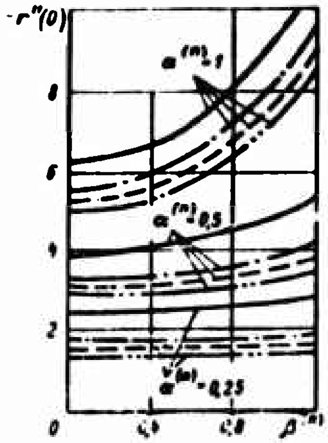

Fig. 8.4. —— n = 1; -·-·-·- n = 2; --- n = 3; -·-·- n = 4.

1) with a change in the number of stages n from one to four the second derivative of the envelope at constant values of $\alpha^{(n)}$ and $\beta^{(n)}$ decreases insignificantly;

2) the nature of the dependence of the second derivative on coefficients $\alpha^{(n)}$ and $\beta^{(n)}$ is practically the same at different numbers of stages; as the band coefficient decreases the magnitude of the second derivative decreases almost in proportion to the decrease in $\alpha^{(n)}$; for $\alpha^{(n)} \leq 0.3$ the second derivative is virtually independent of the coefficient of shift in resonance frequency, at least when $|\beta^{(n)}| \leq 1.3$; when $\alpha^{(n)} \geq 0.5$ the second derivative increases with an increase in the coefficient of shift in resonance frequency.

4. Envelope Parameters

Let us find the width of the main lobe and the correlation time of the envelope of the correlation function of the filtered signal. The method of analysis is similar to that used in the preceding chapter. The calculations were made on the electronic computer. Figures 8.5, 8.6, and 8.7 show the dependence of width θ_{m} and correlation time θ_k of the envelope on the passband coefficient for several values of the resonance frequency shift coefficient for 1 and 4 stages. From these figures it follows that the width of the main lobe and correlation time are basically determined by the passband coefficient of the filter. Maladjustment of the filter at a high coefficient of shift in resonance frequency $|\beta^{(n)}| \leq 1.2$ does not cause substantial impairment in the correlation properties of the filtered phase-manipulated signal. Neither does a change in the number of stages n = 1-4 (i.e., the effect of

the shape of the amplitude-frequency characteristic) have a substantial effect on the width and correlation time of the envelope.

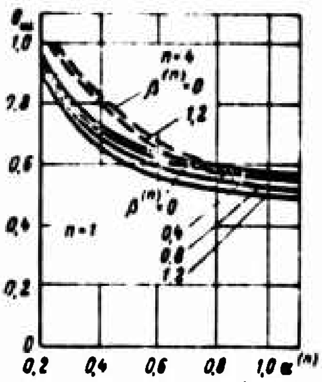

Fig. 8.5. Envelope width.

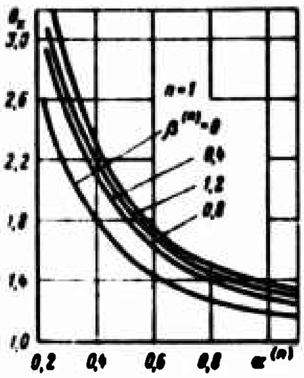

Fig. 8.6. Correlation time (for n = 1).

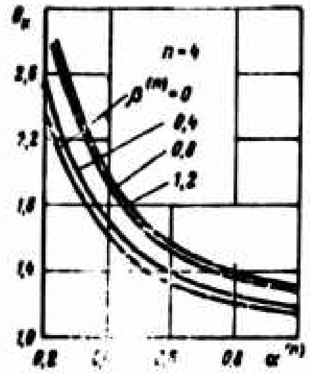

Fig. 8.7. Correlation time (for n = 4).

8.3. EFFECT OF NONIDENTICAL CHARACTERISTICS OF LINEAR UNITS ON SYSTEMATIC ERROR IN MEASURING DELAY

Nonidentical filter parameters in the channels of a correlation system result in different values for group delay times in the signals and are the source of error in measuring delay time. The maximum cross-correlation function of linear filter outputs is shifted in relation to the case of completely identical parameters in these filters. Let us estimate the effect of filter parameters on the magnitude of systematic error in measuring delay.

1. Nonidentical Passbands

Let us assume that the coefficients of shift in resonance frequency are equal to zero. We will introduce dependences plotted on the basis of the formulas derived in § 8.1.

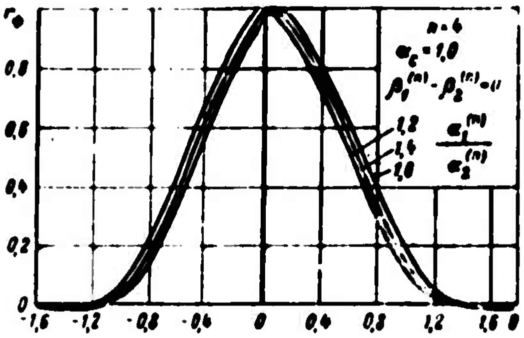

Fig. 8.8. Envelope of cross-correlation function in the case of nonidentical passbands of filters.

In Fig. 8.8 we see the envelopes of the normalized cross-correlation function of phase-manipulated signals for several nonidentical passband values $\alpha_1^{(n)}/\alpha_2^{(n)}$, the mean arithmetic value of the passband coefficient $\alpha_c = 0.5(\alpha_1^{(n)} + \alpha_2^{(n)}) = 1.0$, and a stage number of n = 4. Characteristic is the presence of a shift in the envelope along the x-axis. The greater the nonidentity of the bands, the greater this shift. If the passband of filter Φ_1 is greater than that of Φ_2, i.e., $\alpha_1^{(n)}/\alpha_2^{(n)} > 1$, then the envelope maximum of the cross-correlation function is observed at additional delay in the channel of filter Φ_1. Analysis of many curves plotted for different values of parameters n, $\alpha_1^{(n)}$, and $\alpha_2^{(n)}$ show that at least for a stage number of n = 1-4, a mean arithmetic passband of $\alpha_c = 0.5(\alpha_1^{(n)} + \alpha_2^{(n)}) \geq 0.25$, and a change in the nonidentity of the passbands of $\alpha_1^{(n)}/\alpha_2^{(n)} = 0.6$-$1.6$, the shape of the envelope of the cross-correlation function is virtually unchanged and remains as the envelope of the autocorrelation function of a phase-manipulated signal when $\alpha^{(n)} = \alpha_c$.

In order to find the shift in the envelope maximum or the systematic error in measuring signal delay, which is the same thing, the curves of Figs. 8.9-8.12 were plotted to determine this delay from the moment that the envelope maximum is reached. Shown here is the dependence of the relative time of shift in the maximum θ_M on the nonidentity of the pass bands $\alpha_1^{(n)}/\alpha_2^{(n)}$ for three values of the mean arithmetic passband coefficient α_c = 1.0, 0.5, 0.25 and at a stage number of n = 1-4.

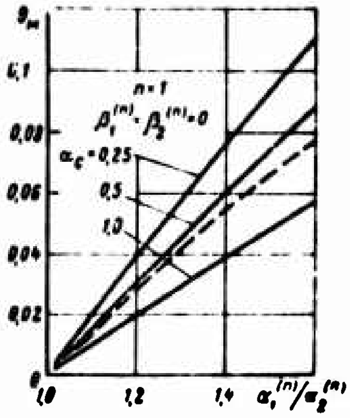

Fig. 8.9. Relative shift in envelope maximum.

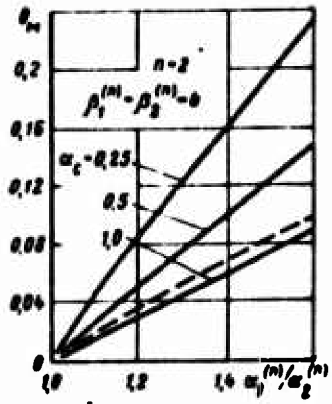

Fig. 8.10. Relative shift in envelope maximum.

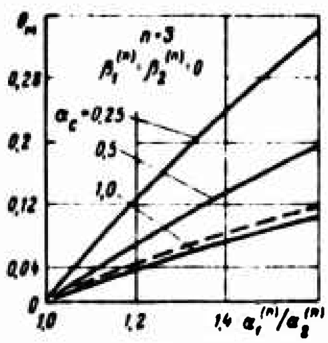

Fig. 8.11. Relative shift in envelope maximum.

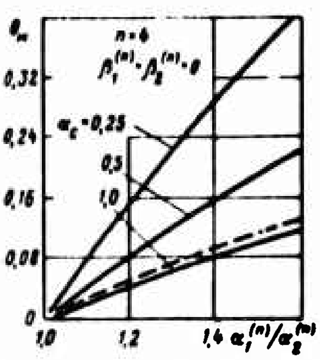

Fig. 8.12. Relative shift in envelope maximum.

From the figures it follows that the shift in the envelope is almost proportional to the nonidentity of the passbands $\alpha_1^{(n)}/\alpha_2^{(n)}$ and grows as the number of stages n increases and as the mean arithmetic band coefficient decreases. The curves are plotted for a passband nonidentity of $_1^{(n)}/\alpha_2^{(n)} > 1$ [sic]. In this case, where $\alpha_1^{(n)}/\alpha_2^{(n)} < 1$, and, consequently, the passband of filter Φ_2 is greater than Φ_1, the envelope maximum of the cross-correlation function will be observed at a certain additional delay in the channel of filter Φ_2. The magnitude of this additional delay can be found with the same diagrams, although the band nonidentity will have to take the reverse value, i.e., $\alpha_2^{(n)}/\alpha_1^{(n)}$.

2. Relative Shift in Resonance Frequency of Filters

Let us examine the effect of a shift in the resonance frequency of filter Φ_1 on the envelope of the cross-correlation function in a case where the passbands of filters Φ_1 and Φ_2 are the same, i.e., $\alpha_1^{(n)} = \alpha_2^{(n)} = a^{(n)}$ and where a shift in the resonance frequency filter Φ_2 is absent, i.e., $\beta_2^{(n)} = 0$.

Figure 8.13 shows envelopes of the normalized cross-correlation function of phase-manipulated signals in the presence of a shift in the resonance frequency of filter Φ_1 (sequence length N = 127). A comparison of curves corresponding to the absence and presence of a shift in resonance frequency $\beta_1^{(n)} = 0$ and $\beta_1^{(n)} = 1.2$ leads to the following conclusions:

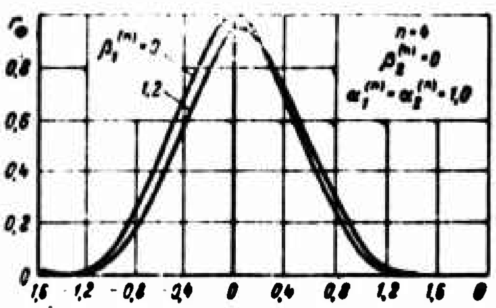

Fig. 8.13. Envelope of cross-correlation function with shift in resonance frequencies of filters.

1) As the shift in the resonance frequency of one filter appears, the envelope maximum of the cross-correlation function shifts along the axis of delay θ and the magnitude of the maximum falls;

2) in the case of a shift in resonance frequency correlation time increases somewhat, although even at high degrees of maladjustment $|\beta_1^{(n)}| = 1.2$ this increase constitutes no more than 30%;

3) additional plottings show that in the presence of a shift in the resonance frequency of one filter the shape of the envelope of the cross-correlation function changes slightly, at least for

$\alpha^{(n)}$ 0.25; $|\beta_1^{(n)}| \leq 1.2$; $n = 1 \div 4$.

Systematic error in measuring the delay of a phase-manipulated signal for the studied case is shown in Figs. 8.14 and 8.15 as a function of the coefficient of maladjustment $\beta_1^{(n)}$ at mean arithmetic band coefficient vuaues of $\alpha_c = \alpha_1^{(n)} = \alpha_2^{(n)} = 0.25$, 0.5, 1.0, and for n = 1 and 4.

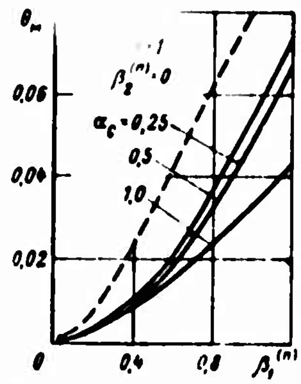

Fig. 8.14. Relative shift in envelope maximum.

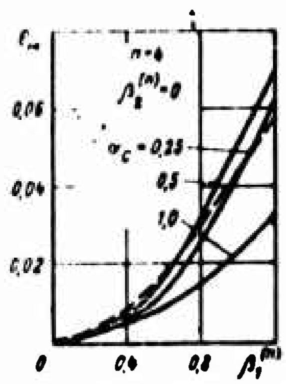

Fig. 8.15. Relative shift in envelope maximum.

From an analysis of the curves it follows that systematic error in measuring delay in the presence of a shift in the resonance frequency of the filter of nee channel relative to the second has a weak dependence on the number of stages at given parameters of $\alpha^{(n)}$ and $\beta_1^{(n)}$. The same systematic error value is also low when $|\beta_1^{(n)}| \leq 1.2$; $\alpha_c \geq 0.25$ is a quantity of less than 7% of the length of the clock interval of the pseudorandom sequence.

The graphic dependences shown above were calculated for pseudorandom sequences of N = 127 cycles. The obtained results can also be used for phase-manipulated signals of greater length. From relationships (8.10)-(8.12) it follows that in the case of a longer sequence, when $N + 1/N \approx 1$, the following quantities will not depend on the length of the sequence at a given cycle [clock] value t_0:

1) the width of the envelope of the cross-correlation function,

2) systematic error in measuring delay of a phase-manipulated signal.

Correlation time depends essentially on N and is weakly dependent on the number of stages n and on shift coefficient $\beta^{(n)}$. A rough value for correlation time can be obtained from (7.31), where instead of α we will substitute the value of the mean arithmetic band coefficient α_c.

8.4. SYSTEMATIC ERROR IN MEASURING DELAY FOR WIDE FILTERS IN THE CHANNELS OF CORRELATION SYSTEM

Error in measuring delay will in a number of cases occur when the passband of the filters in the channels of the correlation system are considerably broader than the frequency band occupied by the spectrum of the signal. These errors are not great, although analyzing and calculating them is valuable in designing correlation systems intended for exact measurements. It is assumed that nonlinear distortions in the signals are absent and that the passbands of the linear filters in the channels Φ_1 and Φ_2 are so great that within the limits of the main part of the energy spectrum of the phase-manipulated signal the amplitude-frequency and phase-frequency characteristic can be represented with sufficient approximation by the first two terms of expansion in the Taylor series, i.e.,

$$K_p(f) \approx K_p(f_c) + K'_p(f_c)(f - f_c),$$
$$\Phi_p(f) \approx \Phi_p(f_c) + \Phi'_p(f_c)(f - f_c),$$

where p = 1, 2.

Let us substitute these relationships in the formulas which determine the correlation coefficient (8.1). We will substitute the variable $x = f - f_c$ and, considering the narrow-band nature of the signals, consider the lower limit of $-f_c$ to be equal to $-\infty$. Then

$$r_{12}(\tau) = \frac{\cos[2\pi f_c\tau + \Phi_1(f_c) - \Phi_2(f_c)]}{\sigma_1\sigma_2}\int_{-\infty}^{\infty} S(f)[K_1(f_c)K_2(f_c) +$$

$$+ K'_1(f_c)K'_2(f_c)f^2]\cos 2\pi f\left[\tau + \frac{\Phi'_1(f_c) - \Phi'_2(f_c)}{2\pi}\right]df -$$

$$- \frac{\sin[2\pi f_c\tau + \Phi_1(f_c) - \Phi_2(f_c)]}{\sigma_1\sigma_2}\int_{-\infty}^{\infty} S(f)f\left[K'_1(f_c)K_2(f_c) +\right.$$

$$\left. + K_1(f_c)K'_2(f_c)\right]\sin 2\pi f\left[\tau + \frac{\Phi'_1(f_c) - \Phi'_2(f_c)}{2\pi}\right]df.$$

The expression for $r_{21}(\tau)$ is obtained from $r_{12}(\tau)$ by a mutual inversion of the indices of Φ_1 and Φ_2. Since the energy spectrum and the correlation function of the pseudorandom sequence are related by the Wiener-Khinchin transform, then the following relationships are quite obvious

$$r'(\tau) = -2\pi\int_{-\infty}^{\infty} fS(f)\sin 2\pi f\tau\,df.$$

$$r''(\tau) = -4\pi^2\int_{-\infty}^{\infty} f^2S(f)\cos 2\pi f\tau\,df.$$

In this case the normalized cross-correlation function, which is written in the form of the product of the envelope times the cosinusoidal function, will have the form of

$$r_{12}(\tau) = A\cos[2\pi f_c\tau + \Phi_1(f_c) - \Phi_2(f_c) - \vartheta(\tau)], \qquad (8.14)$$

where

$$A = \frac{1}{\sigma_1\sigma_2}\left\|\left[\left\{K_1(f_c)K_2(f_c)\,r\left[\tau + \frac{\Phi'_1(f_c) - \Phi'_2(f_c)}{2\pi}\right] - \right.\right.\right.$$

$$\left. - \frac{K'_1(f_c)K'_2(f_c)}{4\pi^2}\,r''\left[\tau + \frac{\Phi'_1(f_c) - \Phi'_2(f_c)}{2\pi}\right]\right\}^2 +$$

$$+ \left\{\frac{K'_1(f_c)K_2(f_c) + K_1(f_c)K'_2(f_c)}{2\pi}\,r'\left[\tau +\right.\right.$$

$$\left.\left.\left. + \frac{\Phi'_1(f_c) - \Phi'_2(f_c)}{2\pi}\right]\right\}^2\right\|^{1/2}.$$

$$\vartheta(\tau) = \operatorname{arctg} \frac{\frac{1}{2\pi}[K'_1(f_c)K_2(f_c) + K_1(f_c)K'_2(f_c)] \times r'\left[\tau + \frac{\Phi'_1(f_c) - \Phi'_2(f_c)}{2\pi}\right]}{K_1(f_c)K_2(f_c) \cdot r\left[\tau + \frac{\Phi'_1(f_c) - \Phi'_2(f_c)}{2\pi}\right] - \frac{K'_1(f_c)K'_2(f_c)}{4\pi} \cdot r''\left[\tau + \frac{\Phi'_1(f_c) - \Phi'_2(f_c)}{2\pi}\right]};$$

$$\sigma_1 = \sqrt{K_1^2(f_c) - \frac{r''(0)}{4\pi^2}[K'_1(f_c)]^2};$$

$$\sigma_2 = \sqrt{K_2^2(f_c) - \frac{r''(0)}{4\pi^2}[K'_2(f_c)]^2}.$$

In analyzing the cross-correlation function from the relationships of (8.14) we must calculate the values of the autocorrelation function of the pseudorandom sequence and its first two derivatives. The autocorrelation function of a binary pseudorandom sequence in the range of the main lobe has a triangular nature and, consequently, its first delay derivative will have an undetermined magnitude at the break points, while the second derivative will have the form of the delta-function. However, it should be kept in mind that this is valid for sequence pulse fronts of infinitely small duration. In practice the pulse fronts of a sequence shaped by a shift register have a finite magnitude. The finite magnitude of the fronts can be considered by introducing at the output of the source of the ideal pseudorandom sequence a RC type of low-frequency filter with a corresponding band coefficient α, which depends on the properties of the cells [bits] of the forming register. Since the amplitude-frequency characteristic of the RC-filter is the same as that of a single oscillatory circuit with a resonance frequency equal to zero, then in analyzing the cross-correlation function from (8.14) we must use the expression for the envelope of the phase-manipulated signal which has passed through a single oscillatory circuit. It is quite obvious that in view of the wide-band nature of the linear filters, the envelope for relationship (8.14) must be close in form to the envelope of the autocorrelation function of a phase-manipulated signal. However, as follows from (8.14), in the case of nonidentical parameters in these filters there is a shift in the position of the envelope maximum, which is equal to

$$\tau_{\text{M}} = \frac{1}{2\pi}\left[\Psi'_2(f_c) - \Phi_1(f_c)\right].$$

or, if we shift to relative delay time introduced earlier,

$$\theta_{\text{M}} = \frac{\tau_{\text{M}}}{t_0} = \frac{1}{2\pi t_0}\left[\Psi'_2(f_c) - \Psi'_1(f_c)\right]. \qquad (8.15)$$

For a case where the linear filters of the correlation system are n-stage resonance amplifiers tuned to the same frequency, the expressions for the phase-frequency characteristics will have the form of

$$\Psi_p = n \cdot \operatorname{arctg} \frac{2\varepsilon_n (f - f_p)}{B_p^{(n)}},$$

where p = 1, 2, and systematic error in measuring the delay of phase-manipulated signals will be equal to

$$\theta_{\text{M}} = -\frac{n\varepsilon_n}{2\pi}\left\{\frac{1}{\alpha_1^{(n)}\left[1 + (\varepsilon_n \xi_1^{(n)})^2\right]} - \frac{1}{\alpha_2^{(n)}\left[1 + (\varepsilon_n \xi_2^{(n)})^2\right]}\right\}. \qquad (8.16)$$

The dotted curves in Figs. 8.9-9.12 represent systematic error values calculated by means of the asymptotic formula (8.16) for the case of $\alpha_c = 1.0$. By comparing the curves plotted from the approximate formula of (8.16) and calculated by means of exact relationships, it follows that the approximation is sufficient, and that the greater the number of stages, the more satisfactory it will be. Thus, to calculate systematic error in measuring the time shift of two phase-manipulated signals in the case of wide-band filters for which the passband coefficient $\alpha_1^{(n)}$, $\alpha_2^{(n)} \geq 1$-1.5 the asymptotic formula of (8.16) is valid for a sufficient degree of practical accuracy.

In systems for measuring distances with a higher degree of accuracy [64] delay can be further defined at a sufficient signal/noise ratio from the high-frequency filling of the cross-correlation function. The system becomes a phase meter, which

places more rigid demands on the stability of the phase characteristics of the receiving and reference channels. Since delay is registered as before, according to the envelope maximum of the cross-correlation function, then, as follows from (8.14), phase angle $\theta(\tau)=0$. Consequently, the phase shift in the filling of the cross-correlation function is equal to

$$\Delta\varphi = \Phi_1(f_c) - \Phi_2(f_c) = n\,[\operatorname{arctg} \varepsilon_n \beta_1^{(n)} - \operatorname{arctg} \varepsilon_n \beta_2^{(n)}].$$

Obviously in exact delay measurement we are interested in small, permissible phase increments $\Delta\phi$, which are determined by the nonidentity of the phase characteristics of the channels. If we assume that the channels are equivalent from the standpoint of possible change in the phase characteristics and consider that the phase shift acquires its greatest value under a different maladjustment sign, i.e.,

$$\beta_1^{(n)} = -\beta_2^{(n)} = \beta_{доп}^{(n)}.$$

then we write the requirements for the permissible coefficient of shift in the resonance frequencies of linear filters in the form of

$$\beta_{доп}^{(n)} = \frac{\Delta\varphi_{доп}}{n\varepsilon_n}. \qquad (8.17)$$

The conclusions drawn in this chapter are in good agreement with experimentally obtained measurements. The results of experimental verification of the envelope of the cross-correlation function for several filter parameters are presented together with the corresponding theoretical curves in Figs. 8.16 and 8.17. Confirmed are the deductions of the presence of a shift in the maximum of the cross-correlation function in the case of nonidentical parameters in filters Φ_1 and Φ_2 and of a certain increase in correlation time in the case of a shift in the resonance frequency of the filter relative to the central frequency of the phase-manipulated signal which enters its input.

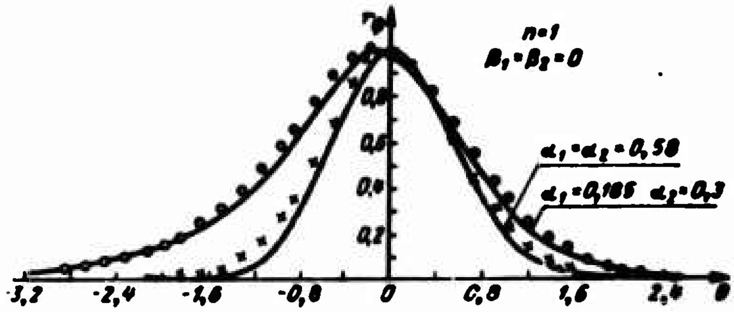

Fig. 8.16. Calculated and experimental dependences of envelope of cross-correlation function of filtered phase-manipulated signals.

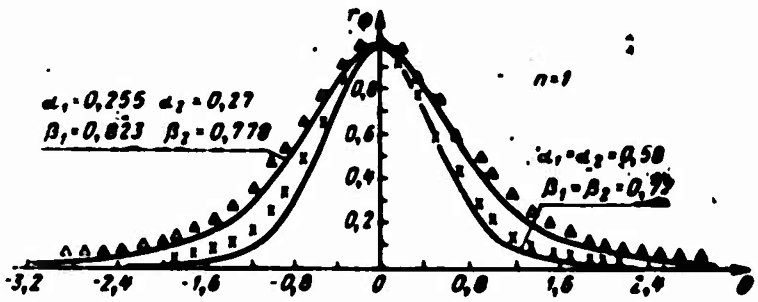

Fig. 8.17. Calculated and experimental dependences of envelope of cross-correlation function of filtered phase-manipulated signals.

APPENDIX

Integral (4.13) is found by using the following calculations for the contour integral [14, p. 356]:

$$\frac{a\Gamma(\nu+1)}{2\pi i}\int_l w^{k-\nu-1} I_m(wP_j)\exp(0.5\sigma_j^2 w^2)\,dw =$$

$$= \frac{a\Gamma(\nu+1)P_j^m}{2^{\frac{m-k+\nu+2}{2}}\,\Gamma\left(1-\frac{m+k-\nu}{2}\right)\sigma_j^{m+k-\nu}} \times$$

$$\times\ {}_1F_1\left(\frac{m+k-\nu}{2};\ m+1;\ -\frac{P_j^2}{2\sigma_j^2}\right),$$

where $I_m(z)$ is a modified Bessel function of the first kind: ${}_1F_1(a;\ b;\ x)$ is a hypergeometric function, determined by the following series:

$${}_1F_1(a;\ b;\ x) = 1 + \frac{ax}{b1!} + \frac{a(a+1)}{b(b+1)2!}x^2 + \ldots$$

If we consider the relationships of (4.10) for particular values of $m = 0$, $k = 0$, $P_j = 0$, then we find the unknown integral:

$$\overline{z(t)} = \frac{a\Gamma(\nu+1)\sigma_\nu^\nu}{2^{1+\frac{\nu}{2}}\,\Gamma\left(1+\frac{\nu}{2}\right)}[\rho(t,0)]^{\frac{\nu}{2}}. \qquad \text{(A.1)}$$

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